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Distributed Medium Access and Opportunistic Scheduling for Ad-Hoc Networks: an Analysis of the Constant Access Time Problem

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***Abstract*—In this paper, we study the problem of medium access and distributed opportunistic scheduling to exploit channel fluctuations for wireless ad-hoc networks. This work focuses on the Constant Access Time (CAT) problem, where the total time duration of channel probing and data transmission is slotted into fixed block length. In particular, we consider the dependence between channel rates at different time instances during the channel probing phase and its impact on the overall system throughput. We first analyze the system performance of the CAT problem under independent channel rate assumption and compare our result to the existing work on the Constant Data Time (CDT) problem. We then propose a protocol to reduce the channel probing costs based on this analysis. We show analytical results of the proposed protocol for systems with a sufficiently large number of users. We compare the system performance of the proposed protocol with that of the original protocol under independent channel rate assumption. We mathematically prove that the proposed protocol improves the system performance.**

I. Introduction

There have been many works on opportunistic scheduling to exploit channel fluctuations for better overall system performance in the past decade. Instead of treating fading as a source of unreliability and trying to mitigate channel fluctuations, fading can be exploited by transmitting information *opportunistically* when and where the channel is strong [1]. Most existing works assume a cellular system model in which a central scheduler tries to optimize the overall system performance by selecting the *on-peak* user for data transmissions [2]–[7]. In contrast, in ad-hoc networks it is necessary to access the wireless medium and schedule data transmissions in a distributed fashion. So far few works have studied the opportunistic scheduling problem in distributed scenarios; examples include rate adaptation with MAC design based on the RTS/CTS handshaking for IEEE 802.11 networks [8]–[10] and channel-aware ALOHA for uplink communications [11]–[13]. However, rate adaptation schemes focus on exploiting temporal opportunities, leaving the distributed medium access problem to the RTS/CTS mechanism [8]–[10]. On the other hand, channel-aware ALOHA associates the probability to access the uplink with channel quality assuming that each user knows its own channel state information (CSI) from the uplink [11]–[13]. These works ignore the overhead due to distributed medium access for wireless networks. In fact, these costs should be considered in the design of the opportunistic scheme in order to fully exploit the channel fluctuations in the network.

One type of distributed opportunistic scheduling (DOS) problems is studied in [14] and [15], in which M links contend the shared wireless medium and schedule data transmissions using only local information in an ad-hoc network. In [14] and [15], the transmitter has no knowledge of other links' channel conditions, and even its own channel condition is not available before a successful probing. The link condition corresponding to one successful probing can either be good or poor due to channel fluctuations. In each round of channel probing, the winner decides on whether or not to send data over the channel. If the winner gives up the current opportunity, all links re-contend the wireless medium again, in the hope that a link with better channel condition can utilize the channel after re-contention. The purpose of this procedure is to optimize the overall system performance. It is shown in [14] and [15] that the decision on further channel probing or data transmission is based only on local channel condition, and the optimal strategy is a threshold policy.

In [14] and [15] it is assumed that the winners' channel rates are independent at any time instance during the channel probing phase. In this paper, we consider the possible dependence between the winners' channel rates at different time instances during the probing phase and its impact on the system throughput. Unlike the constant data time (CDT) problem studied in [14] and [15], we focus on the constant access time (CAT) problem, where the total duration of the channel probing and data transmission is divided into fixed block lengths. We first analyze the system performance of the CAT problem under the independent channel rate assumption. Motivated by the analytical results, we propose a protocol to reduce the channel probing costs. We derive analytical results of the proposed protocol for systems with a sufficiently large number of users. We compare the system performance of the proposed protocol with that of the distributed opportunistic scheduling scheme in [14] and [15] under ideal independent channel rate assumption. We show that the proposed protocol improves the system performance as a result of the reduced probing costs.

This paper is organized as follows. After introducing our system model in Section II, we first analyze the network throughput of the CAT problem under independent channel rate assumption in Section III. We then present a new protocol for opportunistic scheduling in Section IV and show its performance improvement in Section V. Finally we conclude this paper in Section VI.

II. System Model

Similar to [14] and [15], we assume that there are M links sharing the wireless medium in an ad-hoc network without any centralized coordinator. To access the wireless medium, all links have to probe first. Suppose that the links adopt a fixed probing duration τ. A collision channel model is assumed, where a user wins the channel if and only if no other links are probing at the same time. If link m probes the channel with a fixed probability $p^{(m)}$, the duration of the n-th round of channel probing is

$$T_n = \tau \cdot K_n, \tag{1}$$

where K_n is the number of mini-slots probed before the channel is won by the winner. Hence K_n has a geometric distribution with the successful probing probability parameter

$$p_s = \sum_{m=1}^{M} p^{(m)} \prod_{j \neq m} (1 - p^{(j)}). \tag{2}$$

At the end of the n-th round, the winner s_n has an option to send data through the wireless channel at rate R_n, or to give up. Here we denote the channel rate of link m as $R^{(m)}$, and the channel rate of the winner in the n-th round as R_n.

To opportunistically schedule transmissions in a distributed fashion, all M links cooperate to optimize the average network throughput. At the end of the n-th round, s_n makes a decision on whether or not to utilize the channel for data transmission, where s_n sends data over the channel only when R_n is satisfactory. If s_n gives up the opportunity, all links re-contend the wireless medium. This procedure repeats until a link finally utilizes the channel for data transmission. The goal is that all links cooperate to make the channel accessible by someone with sastisfactory channel quality. This problem can be modeled as an optimal stopping problem [16], [17]. In the n-th round, the winner s_n observes the probing duration T_n and the available transmission rate R_n. Hence the sequence of σ-fields [16], [17] can be written as

$$\mathcal{F}_n = \{R_1, T_1;\, R_2, T_2;\, \ldots;\, R_n, T_n\}. \tag{3}$$

At time n, based on $\mathcal{F}_n$, s_n makes a decision on whether to stop or not to maximize the system throughput.

In [14] and [15], the winners' channel rates R_n are explicitly assumed to be independent during the channel probing phase. On the other hand, it is also assumed that R_n can be *locked* for a constant duration T for data transmission under the CDT model. In this paper, we consider the dependence of R_n at different rounds, n and its impact on the system throughput. For this purpose we adopt the constant access time (CAT) model [18], where the total duration of channel probing and data transmission is a constant, i.e. $T_p + T_d = T$. Notice that the duration of channel probing is a random variable depending on the stopping time N, i.e. $T_{p,N} = \sum_{i=1}^{N} T_i$. Furthermore, we assume that the channel has a block fading with block length T, and the channel rates $R^{(m)}$ are *i.i.d.* with a CDF $F_R(r)$ for all $m = 1, \ldots, M$. Notice this does not necessarily mean the independence of winners' rates R_n, since some link $\tilde{m}$ may win the channel for multiple times during the channel probing phase.

III. The CAT Problem under Independence Assumption

In this section, we analyze the average network throughput of the CAT problem assuming winners' channel rates R_n are independent for $n = 1, 2, \ldots$. We compare the average network throughput of CAT problem with that of the CDT problem shown in [14] and [15].

For the CAT problem, at the end of the n-th round of channel probing, the payoff is

$$Y_n = \frac{R_n(T - \sum_{i=1}^{n} T_i)}{T} \tag{4}$$

if the winner s_n decides to utilize the channel, and is equal to 0 otherwise. If this procedure is repeated L times independently, the decision making process can be described as

$$Y^* = \max_{N} \frac{E[R_N \cdot (T - \sum_{i=1}^{N} T_i)]}{T}. \tag{5}$$

We make the following assumptions:

(S1) The channel rates R_n can only take values in the interval $(0, +\infty)$, where $n = 1, 2, \ldots$;

(S2) The probing duration τ is much smaller compared to the block length, i.e. $\tau \ll T$.

Theorem 1: Assume that the winners' channel rates R_n are *i.i.d.* for $n = 1, 2, \ldots$. The average network throughput of the CAT problem λ^*_{CAT} is the solution to

$$E[1 - \frac{\lambda}{R_1}]^+ = \frac{\tau}{T p_s}, \tag{6}$$

where $E[\cdot]^+$ is defined as $E[X]^+ = E[\max(X, 0)]$. The optimal stopping rule N^* is

$$N^* = \min\{n \geq 1 : R_n \geq \lambda^*_n \cdot \frac{T}{T - \tau \sum_{i=1}^{n} K_i}\}, \tag{7}$$

where λ^*_n is the solution to

$$E[1 - \frac{\tau}{T}(\sum_{i=1}^{n} K_i - K_{n+1}) - \frac{\lambda}{R_n}]^+ = \frac{\tau}{T p_s}. \tag{8}$$

Here $\{K_n, n = 1, 2, \ldots\}$ are geometric *i.i.d.* random variables with parameter p_s.

Proof: Under assumption (S1), we rewrite the system payoff (4) at time n as

$$Y_n = \frac{T - \sum_{i=1}^{n} T_i}{T / R_n}.$$

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Hence this problem can be solved as a maximal rate of return problem [16], [17]. For a fixed rate λ, we define a new payoff at time n as

$$Z_n = T - \sum_{i=1}^{n} T_i - T \cdot \frac{\lambda}{R_n}. \tag{9}$$

To show the existence of the optimal stopping rule, we first notice that $E\{\sup_n Z_n\} < T < \infty$. On the other hand, we can easily see $\limsup_{n\to\infty} Z_n \to -\infty$ and $Z_n \to -\infty$ *a.s.*. Therefore,

$$\limsup_{n\to\infty} Z_n \to Z_\infty \quad a.s.$$

Hence an optimal stopping rule exists and can be given by the principle of optimality [16], [17].

Under the assumption (S2) this problem can be treated as an infinite horizon problem. To calculate the optimal payoff based on observations up to time n, we write the payoff at time $n+l$ as

$$Z_{n+l} = T - \tau\sum_{i=1}^{n} K_i - \tau\sum_{i=n+1}^{n+l} K_i - T \cdot \frac{\lambda}{R_{n+l}} \tag{10}$$

$\forall l \in \mathbb{N}$. The system payoff at time $n+l+1$ is

$$Z_{n+l+1} = T - \tau\sum_{i=1}^{n} K_i - \tau K_{n+1} - \tau\sum_{i=n+2}^{n+l+1} K_i - T \cdot \frac{\lambda}{R_{n+l+1}}.$$

Notice that R_n and T_n are independent and R_n and T_n are *i.i.d.* respectively. Therefore, this is the same procedure as in (10) except for an additional cost at time $n+1$. We can write the optimality condition as

$$V_n^*(\lambda) = E[\max\{T - \tau\sum_{i=1}^{n} K_i - T\frac{\lambda}{R_n}, V_n^*(\lambda) - \tau K_{n+1}\}].$$

The optimal payoff λ^* that maximizes the rate of return should yield $V_n^*(\lambda^*) = 0$, i.e.

$$E[\max\{T - \tau\sum_{i=1}^{n} K_i - T\frac{\lambda}{R_n}, -\tau K_{n+1}\}] = 0,$$

which can be written as

$$E[T - \tau(\sum_{i=1}^{n} K_i - K_{n+1}) - T\frac{\lambda}{R_n}]^+ = \tau E[K_{n+1}].$$

Notice we have $E[K_{n+1}] = E[K_1] = 1/p_s$. Substituting it into the above equation and dividing by T both sides yields (8). Denote the optimal solution of (8) as λ_n^*. We can then derive the optimal stopping rule as

$$N^* = \min\{n \geq 1 : T - \tau\sum_{i=1}^{n} K_i - T\frac{\lambda_n^*}{R_n} \geq V_n^*(\lambda_n^*)\}$$
$$= \min\{n \geq 1 : R_n \geq \lambda_n^* \cdot \frac{T}{T - \tau\sum_{i=1}^{n} K_i}\}.$$

To get the overall optimal system throughput, letting $n = 1$ in (8) yields

$$E[1 - \frac{\tau}{T}(K_1 - K_2) - \frac{\lambda}{R_1}]^+ = \frac{\tau}{Tp_s}.$$

Since K_1 and K_2 are *i.i.d.*, and $\tau \ll T$, we can approximate the equation as (6). The solution to (6) is then the optimal system throughput, i.e. $\lambda_{CAT}^* = \lambda_1^*$.

The uniqueness of the solution λ_n^* can be verified in a similar way as [14] and [15], for $n = 1, 2, \ldots$ respectively. ■

We compare the system throughput of the CAT problem with that of the CDT problem shown in [14] and [15]. We observe a smaller overall system throughput in the CAT problem due to its more practical assumption.

Proposition 1: Assume λ_{CAT}^* and λ_{CDT}^* are the optimal average network throughput of the CAT and CDT problem respectively, then

$$\lambda_{CAT}^* < \lambda_{CDT}^* \tag{11}$$

Proof: Since R_n are *i.i.d.* with a CDF $F_R(r)$, (6) can be written as

$$\frac{\tau}{Tp_s} = \int_\Omega (1 - \frac{\lambda_{CAT}}{r}) dF_R(r),$$

where $\Omega = \{r | 1 - \frac{\lambda_{CAT}}{r} > 0\} = \{r | r > \lambda_{CAT}\}$. Since $r > \lambda_{CAT}$ for all $r \in \Omega$, we have the basic inequality

$$\frac{\lambda_{CAT}}{r} + \frac{r}{\lambda_{CAT}} > 2.$$

Writing this inequality as $0 < 1 - \frac{\lambda_{CAT}}{r} < \frac{r}{\lambda_{CAT}} - 1$ and substituting it into the integration yields

$$\frac{\tau}{Tp_s} < \int_{\lambda_{CAT}}^{+\infty} (\frac{r}{\lambda_{CAT}} - 1) dF_R(r) = E[\frac{R_1}{\lambda_{CAT}} - 1]^+$$

For the CDT problem, the average network throughput λ_{CDT}^* is the solution to the following equation [14], [15]:

$$E[R_1 - \lambda_{CDT}]^+ = \frac{\tau\lambda_{CDT}}{p_s T},$$

which can be rewritten as

$$E[\frac{R_1}{\lambda_{CDT}} - 1]^+ = \frac{\tau}{p_s T}.$$

Hence we have

$$E[\frac{R_1}{\lambda_{CAT}} - 1]^+ > E[\frac{R_1}{\lambda_{CDT}} - 1]^+, \tag{12}$$

which immediately yields the inequality (11). ■

IV. Protocol Design

In this section, we design a protocol for improved distributed opportunistic scheduling, which is motivated from the system throughput of the CAT problem (6).

From (6), we can see that the optimal throughput λ^* decreases as the successful probing probability p_s decreases. In order to see how the number of links M affects λ^*, we consider a special case where all M users probe with the same probability p. In this case, the successful probing probability in (2) can be simplified to

$$p_s = M \cdot p(1-p)^{M-1}.$$

Algorithm 1 Distributed Opportunistic Scheduling with Reduced Probing Cost

Initially a subset of users $\mathcal{M}$ is assigned to the channel;
for every user $m \in \mathcal{M}$ **do**
  m probes the channel with a fixed probability $p^{(m)}$;
  if m wins the channel **then**
    m makes a decision on whether to send data on the channel or not;
    if m decides to utilize the channel **then**
      m sends data through the channel for duration $T - \sum_{i=1}^{n} T_i$, where n is the current index of channel probing;
    else
      Delete m from the list $\mathcal{M}$;
    end if
  end if
end for

Taking the first-order derivative

$$\frac{\partial p_s}{\partial p} = M \cdot (1-p)^{M-2} \cdot (1 - Mp).$$

and writing the optimality condition yields that p_s is maximized at $p^* = 1/M$. This corresponds to a scenario where exactly $Mp^* = 1$ user probing the channel on average at any time. The maximum successful probing probability is then

$$p_s = \frac{1}{1-\frac{1}{M}} \cdot (1 - \frac{1}{M})^M \tag{13}$$

for $M \geq 2$. It is a decreasing function of M. Hence the optimal throughput λ^* is a decreasing function as the number of links M increases. From the perspective of channel probing costs, a smaller M is preferred for better system performance. However, a sufficiently large M is still needed from the perspective of multiuser diversity.

On the other hand, for any given m the channel rate $R^{(m)}$ does not change within a block length T. Hence in the n-th round of channel probing, if winner s_n decides to give up the opportunity for data transmission, the same decision will be repeated for all future $\tilde{n} > n$ when the channel is won by s_n again. This is because utilizing the channel at time $\tilde{n}$ will not increase the payoff, i.e.

$$\frac{R_{\tilde{n}} \cdot T_{d,\tilde{n}}}{T} < \frac{R_n \cdot T_{d,n}}{T},$$

where we have used $R_{\tilde{n}} = R_n = R^{(s_n)}$. It implies that if link m ever decides to send data through the channel, it should happen when m wins the channel for the first time. After that m should not contend the channel at all, since: i) it affects the successful probability p_s, and ii) m will repeat the same decision (i.e. to give up the opportunity). This idea immediately leads to an improved protocol shown as Algorithm 1.

In Algorithm 1, the cardinality of the set of active probing links is decreasing in the channel probing phase, i.e.

$$M_n \triangleq \|\mathcal{M}_n\| = M_1 - n + 1, \tag{14}$$

where M_1 is the initially assigned number of links. Denote $p_{s,n}$ as the successful probing probability in the n-th round of channel probing. Then,

$$p_{s,1} < p_{s,2} < \ldots < p_{s,n} < \ldots, \tag{15}$$

and

$$T_1 > T_2 > \ldots > T_n > \ldots, \tag{16}$$

where $T_n = \tau \cdot K_n$, and K_n has a geometric distribution with parameter $p_{s,n}$. Reducing the size of the set of active probing links $\mathcal{M}_n$ is essential in Algorithm 1. It not only reduces the probing cost, but also ensures that the winner s_n is different in each round of the channel probing. Hence in Algorithm 1 winners' rate R_n are independent for $n = 1, 2, \ldots$. In contrast, the independence of winners' rates are explicitly assumed in [14] and [15], since the set of active probing users $\mathcal{M}_n$ does not change there.

V. Performance Analysis

In this section, we analyze the proposed protocol and show its performance improvement compared to the scheme in Section III.

Compared to the CAT problem with independent channel rate assumption, we notice that for $n = 1, 2, \ldots$, K_n are still independent but with different distributions. In this case, it is difficult to analyze its performance for arbitrarily $p^{(m)}$ for $m = 1, \ldots, M$, even though K_n has a geometric distribution for all n. This is because in Algorithm 1 the set of active probing users $\mathcal{M}_n$ is time varying. We make the following additional assumptions:

(S3) Each user m in the set $\mathcal{M}_n$ is probing the wireless medium with a fixed probability $p^{(m)} = p$;
(S4) The initial number of active probing links M_1 is large enough.

With the assumption (S3), the successful probing probability in the n-th round can be written as

$$p_{s,n} = M_n p (1-p)^{M_n - 1}, \tag{17}$$

where M_n is the cardinality of the set of the active probing users. Under assumption (S4), the initial number of probing users M_1 is sufficiently large for the optimal stopping problem, i.e. $M_1 \gg E[N]$, where N is the stopping time. Following (14), we can approximate the successful probing probability (17) as

$$\begin{aligned} p_{s,n} &\approx M_1 p (1-p)^{M_n - 1} \\ &= M_1 p (1-p)^{M_1 - n}. \end{aligned} \tag{18}$$

It can also be written in a recursive way as

$$p_{s,n} = (1-p) p_{s,n-1} \tag{19}$$

for $n \geq 2$. We introduce a new parameter as

$$f_n = \frac{p_{s,n}}{p_{s,1}} = (1-p)^{-(n-1)}. \tag{20}$$

Since K_n has a geometric distribution with parameter $p_{s,n}$, its expectation can be written as

$$E[K_n] = \frac{1}{p_{s,n}} = \frac{1}{f_n p_{s,1}} = \frac{1}{f_n} \cdot E[K_1].$$

For each $n \geq 1$, we define a new random variable as

$$\tilde{K}_n = f_n \cdot K_n. \tag{21}$$

In addition, $\tilde{K}_n$ has a geometric distribution with mean $E[\tilde{K}] = f_n E[K_n] = E[K_1]$. Here $\tilde{K}_n$ and K_1 can be considered *equal in distribution* [19]. Since K_n are independent with each other, $\tilde{K}_n$ are also independent. Hence we have a new sequence of *i.i.d.* random variables $\{\tilde{K}_n, n = 1, \ldots\}$.

Theorem 2: Assume $p^{(m)} = p$ for all $m \in \mathcal{M}$ and the initial number of users M_1 is large enough, the average network throughput λ^*_{CAT} of Algorithm 1 is the solution to

$$E[1 - \frac{\tau}{T}\{K_1 - (1-p)^2 \tilde{K}_2\} - \frac{\lambda}{R_1}]^+ = (1-p)^2 \frac{\tau}{T p_{s,1}}. \tag{22}$$

The optimal stopping rule N^* is the same as (7), except that λ^*_n is the solution to

$$\begin{aligned} &E[1 - \frac{\tau}{T}\{\sum_{i=1}^{n} K_i - (1-p)^{n+1}\tilde{K}_{n+1}\} - \frac{\lambda}{R_n}]^+ \\ &= (1-p)^{n+1} \frac{\tau}{T p_{s,1}}. \end{aligned} \tag{23}$$

Here K_1 and $\{\tilde{K}_{n+1}, n = 1, 2, \ldots\}$ are *i.i.d.* geometric random variables with parameter $p_{s,1}$.

Proof: Compared to the problem in Section III, the major difference here is that K_n are still independent but not identically distributed for $n = 1, 2, \ldots$. We use the same payoff function Z_n as in Section III. The existence of the optimal stopping rule can be verified in the same way as Section III.

To compute the optimal payoff V^*_n based on observations $\mathcal{F}_n$, we again take a look at the payoff at time $n + l$ for all l rounds of observation. Substituting (20) and (21) into (10) yields

$$\begin{aligned} Z_{n+l} = T &- \tau \sum_{i=1}^{n} (1-p)^{i-1} \tilde{K}_i \\ &- \tau \sum_{i=n+1}^{n+l} (1-p)^{i-1} \tilde{K}_i - T \cdot \frac{\lambda}{R_{n+l}}. \end{aligned}$$

The system payoff at time $n + l + 1$ is then

$$\begin{aligned} Z_{n+l+1} = T &- \tau \sum_{i=1}^{n} (1-p)^{i-1} \tilde{K}_i - \tau(1-p)^n \tilde{K}_{n+1} \\ &- [\tau \sum_{i=n+2}^{n+l+1} (1-p)^{i-1} \tilde{K}_i + T \cdot \frac{\lambda}{R_{n+l+1}}]. \end{aligned}$$

Here $\tau \sum_{i=n+2}^{n+l+1} (1-p)^{i-1} \tilde{K}_i + T \cdot \frac{\lambda}{R_{n+l+1}}$ is the recursive part for l rounds of observations since started from time $n+1$. This can be written as

$$\begin{aligned} &\tau \sum_{i=n+2}^{n+l+1} (1-p)^{i-1} \tilde{K}_i + T \cdot \frac{\lambda}{R_{n+l+1}} \\ &= (1-p)\{\tau \sum_{i=n+1}^{n+l} (1-p)^{i-1} \tilde{K}_{i+1} + T \cdot \frac{\lambda}{R_{n+l+1}}\} \\ &\quad + p \cdot T \cdot \frac{\lambda}{R_{n+l+1}}. \end{aligned}$$

For an infinite horizon problem the probing probability p should be reasonably small, otherwise the average number of users on the channel Mp will be much larger than 1, leading to increased probing costs. Hence we can ignore the last term and write it as

$$\begin{aligned} &\tau \sum_{i=n+2}^{n+l+1} (1-p)^{i-1} \tilde{K}_i + T \cdot \frac{\lambda}{R_{n+l+1}} \\ &\approx (1-p)\{\tau \sum_{i=n+1}^{n+l} (1-p)^{i-1} \tilde{K}_{i+1} + T \cdot \frac{\lambda}{R_{n+l+1}}\}. \end{aligned}$$

This equation leads to a recursive way of computing the optimal payoff based on the observations $\mathcal{F}_{n+1}$. We can write the optimality equation as

$$\begin{aligned} V^*_n(\lambda) = E[\max\{&T - \tau \sum_{i=1}^{n} K_i - T\frac{\lambda}{R_n}, \\ &(1-p)(V^*_n(\lambda) - \tau K_{n+1})\}]. \end{aligned}$$

The optimal payoff λ^* that maximizes the rate of return must satisfy $V^*_n(\lambda^*) = 0$. The equation can thus be simplified as

$$E[\max\{T - \tau \sum_{i=1}^{n} K_i - T\frac{\lambda}{R_n}, -(1-p) \cdot \tau K_{n+1}\}] = 0,$$

which can be further written as

$$E[T - \tau\{\sum_{i=1}^{n} K_i - (1-p)K_{n+1}\} - T\frac{\lambda}{R_n}]^+ = \tau(1-p)E[K_{n+1}]. \tag{24}$$

Notice $K_{n+1} = \frac{1}{f_{n+1}} \cdot \tilde{K}_{n+1} = (1-p)^n \tilde{K}_{n+1}$, and $\tilde{K}_{n+1}$ and K_1 are *i.i.d.*. Therefore the left hand of (24) yields

$$\begin{aligned} &E[T - \tau\{\sum_{i=1}^{n} K_i - (1-p)K_{n+1}\} - T\frac{\lambda}{R_n}]^+ \\ &= E[T - \tau\{\sum_{i=1}^{n} K_i - (1-p)^{n+1}\tilde{K}_{n+1}\} - T\frac{\lambda}{R_n}]^+ \end{aligned}$$

and the right hand of (24) yields

$$E[K_{n+1}] = (1-p)^n E[\tilde{K}_{n+1}] = (1-p)^n \frac{\tau}{p_{s,1}}.$$

Substituting these terms into (24) and then dividing both sides by T, results in (23). Suppose the solution of (23) is λ^*_n. The optimal stopping rule N^* can be derived in the same way as in Section III.

To get the overall optimal system throughput, letting $n = 1$ in (23) immediately yields (22). The optimal overall system throughput is the solution to (22), i.e. $\lambda^*_{CAT} = \lambda^*_1$. ■

In the following proposition, we show the proposed protocol improves the average network throughput.

Proposition 2: Denote λ_{CAT}^{*2} as the average network throughput of Algorithm 1, and λ_{CAT}^{*1} as that of the CAT problem with independence assumptions in Section III. The following inequality holds

$$\lambda_{CAT}^{*2} > \lambda_{CAT}^{*1}. \quad (25)$$

Proof: Under assumption (S2), we notice $\frac{\tau}{T} \ll 1$. We can then ignore the second item in the expectation on the left side of (22) and get

$$E[1 - \frac{\lambda}{R_1}]^+ = (1-p)^2 \frac{\tau}{T p_{s,1}}. \quad (26)$$

Notice that $p_{s,1}$ is equal to the p_s in (6). Comparing (26) to (6) results in the inequality (25). ■

VI. Conclusion

In this paper, we studied a medium access and distributed scheduling problem to exploit the channel fluctuations in an opportunistic fashion for wireless ad-hoc networks. We considered the dependence between the channel rates at different time instances during the probing phase and its impact on the average network throughput. For this purpose we adopted the CAT model, where the total time duration of channel probing and data transmission is slotted into fixed-length blocks. We first analyzed the performance of the CAT problem under independent channel rate assumption. We compared the analytical result to that of the existing work on the CDT problem, motivated by which we proposed a new protocol to reduce the channel probing costs. We derived analytical results for the proposed protocol for networks with an infinite number of users. We proved that the proposed protocol improves the overall system performance compared to the distributed opportunistic scheme with the ideal independent channel rate assumption.

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References

[1] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. New York, NY, USA: Cambridge University Press, 2005.

[2] R. Knopp and P. Humblet, “Information capacity and power control in single-cell multiuser communications,” in *Proc. IEEE International Conference on Communications*, Jun. 1995, pp. 331–335.

[3] P. Viswanath, D. Tse, and R. Laroia, “Opportunistic beamforming using dumb antennas,” *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277–1294, Jun. 2002.

[4] S. Borst and P. Whiting, “Dynamic rate control algorithms for HDR throughput optimization,” in *Proc. IEEE INFOCOM*, 2001, pp. 976–985.

[5] R. Agrawal, A. Bedekar, R. J. La, R. Pazhyannur, and V. Subramanian, “Class and channel condition based scheduler for EDGE/GPRS,” in *Proc. SPIE*, 2001, pp. 59–68.

[6] X. Liu, E. Chong, and N. Shroff, “Transmission scheduling for efficient wireless utilization,” in *Proc. IEEE INFOCOM*, 2001, pp. 776–785.

[7] Y. Liu and E. Knightly, “Opportunistic fair scheduling over multiple wireless channels,” in *Proc. IEEE INFOCOM*, 2003, pp. 1106–1115.

[8] G. Holland, N. Vaidya, and P. Bahl, “A rate-adaptive MAC protocol for multi-hop wireless networks,” in *Proc. ACM MobiCom*, 2001, pp. 236–251.

[9] B. Sadeghi, V. Kanodia, A. Sabharwal, and E. Knightly, “Opportunistic media access for multirate ad hoc networks,” in *Proc. ACM MobiCom*, 2002, pp. 24–35.

[10] Z. Ji, Y. Yang, J. Zhou, M. Takai, and R. Bagrodia, “Exploiting medium access diversity in rate adaptive wireless LANs,” in *Proc. ACM MobiCom*, 2004, pp. 345–359.

[11] X. Qin and R. Berry, “Exploiting multiuser diversity for medium access control in wireless networks,” in *Proc. IEEE INFOCOM*, 2003, pp. 1084–1094.

[12] ——, “Opportunistic splitting algorithms for wireless networks,” in *Proc. IEEE INFOCOM*, Mar. 2004, pp. 1662–1672.

[13] S. Adireddy and L. Tong, “Exploiting decentralized channel state information for random access,” *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 537–561, Feb. 2005.

[14] D. Zheng, W. Ge, and J. Zhang, “Distributed opportunistic scheduling for ad-hoc communications: an optimal stopping approach,” in *Proc. ACM MobiHoc*, 2007, pp. 1–10.

[15] ——, “Distributed opportunistic scheduling for ad hoc networks with random access: An optimal stopping approach,” *IEEE Transactions on Information Theory*, vol. 55, no. 1, pp. 205–222, Jan. 2009.

[16] T. S. Ferguson, *Optimal Stopping and Applications*, 2006. [Online]. Available: http://www.math.ucla.edu/~tom/Stopping/Contents.html

[17] A. Shiryayev, *Optimal Stopping Rules*. Springer-Verlag, 1978.

[18] N. B. Chang and M. Liu, “Optimal channel probing and transmission scheduling for opportunistic spectrum access,” in *Proc. ACM MobiCom*, 2007, pp. 27–38.

[19] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. McGraw Hill, 2002.