Rate-Control System with Heterogeneous Time-varying Delays in Broadband Satellite Networks

Hui Zeng, Michael Hadjitheodosiou and John S. Baras
HyNet, Institute for Systems Research, University of Maryland, College Park, MD 20742
Tel: (301)294-4258, Fax: (301)294-5201, Email: {zengh, michalis, baras}@umd.edu

Abstract

In a broadband satellite communications network, the propagation delays are not only significant, but also variable among users due to their different geographical locations and the problem becomes more severe with increasing data rates. We consider rate control algorithms with user feedback in the form of single bits and formulate analytic fluid flow models composed of first-order delay-differential equations. Both single-flow and multi-flow system models are analyzed, with special attention paid to the Mitra-Seery algorithm. The stationary solutions are investigated first. For the fluctuating solutions, their dynamic behavior is analyzed, in terms of amplitude, transient behavior, fairness and adaptability, etc., analytically and numerically. Especially the effects of heterogeneous time-varying delays are investigated. It is shown that with proper parameter design the system can achieve stabling behavior with close to pointwise proportional fairness among flows.

1. Introduction

Real-time rate-based flow control with feedback is broadly used to avoid remote queue overflow by adjusting the variable data rates assigned to all the flows. In a broadband satellite communications network, however, the time associated with the adaptive processes for feedback-based rate control is in the order of the propagation delays, which are not only significant, but also variable among users due to their different geographical locations in many cases. The problem is more severe considering the high speed. Furthermore, when LEO/MEO/HEO or other moving objects are used as source nodes or intermediate nodes, the propagation delays are time-varying. But feedback-based rate control is still very suitable for broad classes of bursty applications, whose bandwidth demands will persist for comparable time durations to the time of adaptive processes. So it is necessary and important to perform the stability and dynamic behavior analysis of such kind of systems.

We focus on a class of rate control algorithms with feedback to the users in the form of single bits within the broadband satellite communications network. The single bit indicates whether the instantaneous queue size at the distant location is beyond a threshold. In our one-hop network model, a number of flows locate in the moving nodes. Every flow is associated with two nonnegative parameters, v_j and σ_j, for flow j. v_j is the minimum bandwidth and σ_j is the nonnegative weight assigned to the flow j to determine its best-effort share of the available bandwidth. A distant queue has the service rate μ and the queue threshold Q_T. It is worth noting that network models with two or more hops can be converted to the combination of one-hop network models.

We utilize the asynchronous and synchronous versions of general algorithms for feedback-based rate control system, based on our one-hop network model, to introduce suitable fluid models with heterogeneously time-varying propagation delays for single and multiple flows, respectively. We then study the stability and the time-varying behavior of the modelled rate-control system with single-flow. We give the conditions for the existence of stationary solutions, prove the convergence and obtain the convergence rate. We also give the bounds of the fluctuation solutions under the condition when the stationary solutions do not exist; their dynamic behavior of fluctuation solutions is analyzed in detail.

We further our study to the multiple-flows fluid model for rate-control systems, using the same methodology as the single-flow fluid model. Fairness and scalability are two important issues in the algorithm design for the multiple flows. We present the stationary solutions, existence conditions and convergence speed for the multi-flow system models. And then for the situations under which the systems only have fluctuating solutions, we analyze the dynamic behavior of rates and queue size in detail. Based on the analytic results, we investigate the effect of delays and parameters in terms of fairness, fluctuation (amplitude, period), transient behavior and adaptability, etc. It has been shown, analytically and in simulations, that with proper parameter design the system can achieve stabling behavior with close to pointwise proportional fairness among flows.

2. Recent work

In this paper, we extend some related work. A network model with large propagation delays in wide-area network was presented in [1], and its dynamics was fully investigated in both analytic way and simulation. Its network model is very similar to our one-hop network model except that both propagation delays and service rate are fixed. A fundamental theory of response-time based adaptations for large propagation delays is developed in [2]. The damping and gain parameters are selected for the delay-differential equations to optimize transient behavior. A basic symmetric algorithm called Mitra-Seery (MS) and its design rules are given in [1], while an asymmetric algorithm called Jacobson-Ramakrishnan-Jain (JRJ) is introduced in [3]. We also draw ideas for the model formulation, fluid models approximation and behavior analysis from [4].

Recent work has been done on studying the similar systems with fixed delays from another point of view [5, 6, 7, 8, 9, 10, 11]. In [5], a single-flow single-resource case with a fixed feedback delay is analyzed and the stability condition is shown under assumptions on the price function. In [9], the sufficient condition is given for the stability of the single-flow single-resource system with fixed delay and more general utility functions. The case with homogeneous fixed round-trip delay and a given utility function is studied in [8] for stability and convergence rate. In [7] a sufficient condition is given for the stability of a single-resource multiple-flows system with fixed queuing delays. The case with time-varying propagation delays is analyzed under an optimization framework in [10], with the delays modelled by Gamma distribution. In [11], heterogeneous time-varying delays are considered for the primal/dual distributed algorithms that solve the network flow optimization problems.

2. Models for a single flow system

Similarly to the work in [1] but with time-varying propagation delays and service rate, we formulate general asynchronous and synchronous versions for feedback-based rate-control systems, and then utilize fluid models to approximate the systems with single flow and multiple flows, respectively.

In this section we focus on the systems with only one flow, which is modelled as follows:

$$\frac{d}{dt}\varphi(t)=\begin{cases}-\Gamma^{+}[\varphi(t)-\nu]+A^{+}u(t) & \text{if } u(t)>0\\ -\Gamma^{-}[\varphi(t)-\nu]+A^{-}u(t) & \text{if } u(t)<0\end{cases} \quad \text{(a)}$$

$$\frac{d}{dt}q(t)=\begin{cases}[\varphi(t-\tau(t))-\mu(t)] & \text{if } q(t)>0\\ [\varphi(t-\tau(t))-\mu(t)]^{+} & \text{if } q(t)=0\end{cases} \quad \text{(b)} \qquad (1)$$

Here, $\varphi(t)$ is the flow rate in term of the throughput of packets at time t. Also $u(t) = \text{sgn}[Q_T - q(t - \tau(t))]$, and $[.]^+ = \max(., 0)$. Γ^+, Γ^-, A^+, A^- are nonnegative parameters. It is difficult to directly solve equation (1) or study its dynamic behavior. So we will start our analysis from two cases: time-varying delay with fixed service rate; fixed delay with time-varying service rate.

2.1 Case 1: time-varying delay with fixed service rate

The model for a single flow with time-varying propagation delay but fixed bandwidth is

$$\frac{d}{dt}\varphi(t)=\begin{cases}-\Gamma^{+}[\varphi(t)-\nu]+A^{+}u(t) & \text{if } u(t)>0\\ -\Gamma^{-}[\varphi(t)-\nu]+A^{-}u(t) & \text{if } u(t)<0\end{cases} \quad \text{(a)}$$

$$\frac{d}{dt}q(t)=\begin{cases}[\varphi(t-\tau(t))-\mu] & \text{if } q(t)>0\\ [\varphi(t-\tau(t))-\mu]^{+} & \text{if } q(t)=0\end{cases} \quad \text{(b)} \qquad (2)$$

In this case, the time-varying propagation delay does not affect the existence of stationary solutions. We have the same results with the (i) and similar results with (ii) of Proposition 3.2 in [1].

Proposition 1: Suppose $\nu + A^+/\Gamma^+ < \mu$. The system in (2) has a stationary solution: $\varphi \equiv \nu + A^+/\Gamma^+$, $q \equiv 0$. Also:

1.1 If $\varphi(t_1) \le \nu + A^+/\Gamma^+$ for any t_1, then $\varphi(t) \le \nu + A^+/\Gamma^+$ for all $t \ge t_1$. If $\varphi(t_0) > \nu + A^+/\Gamma^+$, then there exists t_2, $t_2 > t_0$, s.t. $\varphi(t)$ decreases monotonically when $t_2 > t > t_0$, and $\varphi(t_2) = \nu + A^+/\Gamma^+$.

1.2 Assume $\tau(t) \in S_T \subset R$ and S_T is compact. Denote its bounds as τ_{min} and τ_{max}. There exists t_3, s.t. for all $t \ge t_3$, $q(t) < Q_T$, $\varphi(t) \to \nu + A^+/\Gamma^+$ at the exponential rate Γ^+.

Proof: skipped.

From the proof of Proposition 1, we also have the following remarks when $\nu + A^+/\Gamma^+ < \mu$ holds:

Remark 1: Proposition 1.1 does not require bounded $\tau(t)$. Proposition 1.2 also holds for unbounded $\tau(t)$ if: 1) $0 < \tau(t) < t$ and 2) $(t - \tau(t))$ is (not necessarily monotonically) increasing to $+\infty$. The conditions can be satisfied when the system is base on FIFO and always in connection during the considered time window.

Remark 2: Recall that Γ^+, Γ^-, A^+, A^- are nonnegative parameters. Further assume that Γ^+, $\Gamma^- > 0$. The system in equation (2) is then globally asymptotically stable with exponential rate.

Also we have the property: regardless of whether $\nu + A^+/\Gamma^+ < \mu$ is satisfied or not, for any t_0 and $t > t_0$,

$$|\psi(t)| \le A_M/\Gamma_M + \left[|\psi(t_0)| - A_M/\Gamma_M\right]\cdot \exp[-\Gamma_M(t-t_0)]. \qquad (3)$$

which is very useful for studying the solutions in the fluctuation region where $\nu + A^+/\Gamma^+ < \mu$ does not hold.

2.2 Case 2: fixed delay with time-varying service rate

The model for a single flow with fixed delay but time-varying bandwidth is obtained by setting $\tau(t) = \tau$ in equation (1). Due to space limitation, the model is skipped here. It is not easy to even find the existence of stationary solutions except for the cases under the following conditions.

Proposition 2: Suppose $\nu + A^+/\Gamma^+ < \inf_{t>t_0} \mu(t)$ for a finite time t_0. Then the system has a stationary solution: $\varphi \equiv \nu + A^+/\Gamma^+$ and $q \equiv 0$.

However, it is worth noting that in broadband satellite communication networks, the service rate is slowly time-varying, compared with the time horizon of the system dynamic behavior due to the heterogeneous propagation delay. Hence, in the rest of this paper, we focus on the rate control system in equation (2) which has time-varying delays with fixed service rate, unless stated otherwise.

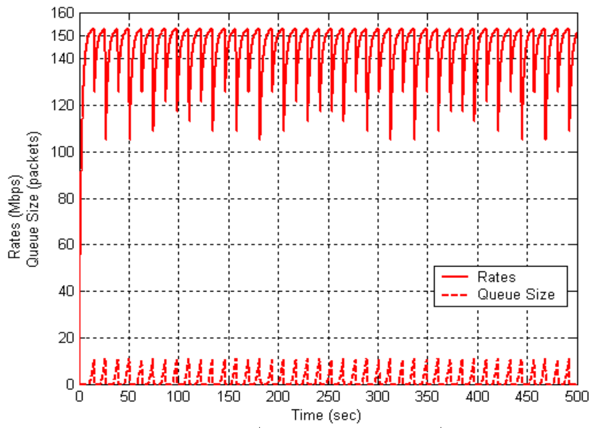
2.3 Solutions in the fluctuation region

In this section we focus on the dynamic system behavior in the fluctuation region and expect that the system has fluctuating solutions with small amplitudes in some sense through careful system designs. The assumptions in this section are listed below:

- $\nu + A^+/\Gamma^+ > \mu$.
- $A^+, A^- \geq 0$ and $\Gamma^+, \Gamma^- > 0$. Consider Mitra-Seery (MS) algorithm where $\Gamma^+ = \Gamma^- = \Gamma$ and $A^+ = A^- = A$.
- $\varphi(t)$ and $q(t)$ are piece-wise differentiable. At time 0, $\varphi(0) = q(0) = 0$.
- $\tau(t) \in S_T \subset R$, where S_T is a compact set with nonnegative lower and upper bounds τ_{min}, τ_{max}.

In the fluctuation region, the demand $\nu + A^+/\Gamma^+$ is even higher than the service rate μ, so unbounded delays may eventually lead to forced dropping of incoming packets in the system. Thus, we consider the case with bounded delays only.

It was originally shown in [1, 12] that the system with fixed delays will have periodic solutions when no stationary solutions exist. However, with time-varying delays, the system does not have equiripple periodic solutions in the fluctuation region, as shown in Figure 1, where as an example we use a sinusoidal function to model time-varying delays. The system has aperiodic fluctuating solutions, with an average rate of 144.2Mbps. Compared with its maximum, the fluctuation of rate is relatively small (< 1/3), and could be even less with careful design of parameters.



μ = 150Mbps, ν = 30Mbps, $A^+ = A^-$ = 48.1Mbps, $\Gamma^+ = \Gamma^-$ = 0.39, Δ = 0.05s, Q_T = 10 packets, $\tau(t)$ = 0.1 + 0.05 · sin($\pi \cdot t$ / 11).

Figure 1: Rate-control system for a single flow with bounded delays

Now we study the dynamic behavior of systems with general time-varying delays in the fluctuation region. Since ν can be absorbed by $\varphi(t)$ and μ in equation (2), we set $\nu = 0$ in the rest of this section.

<u>Phase 1</u>: $t \in (t_0, t_1)$, where $t_0 \equiv \inf\{t \geq 0: \varphi(t) = \mu\}$, $t_1 \equiv \inf\{t \geq t_0: q(t - \tau(t)) = Q_T\}$. We have $\varphi(t_0) = \mu$ and $q(t_0) = 0$. Clearly, in Phase 1, $u(t) = \mathrm{sgn}[Q_T - q(t - \tau(t))] \equiv 1$, $\varphi(t)$ overshoots μ and always increases; $q(t)$ stays positive and increases except a subset of $(t_0, t_0 + \tau_{max})$. Assuming $t_0 + \tau_{max} << t_1$ and ignoring this small transition subset, the governing equations for Phase 1 are

$$\begin{cases} d\varphi(t)/dt = -\Gamma \cdot \varphi(t) + A \\ dq(t)/dt = \varphi(t - \tau(t)) - \mu \end{cases} \quad (4)$$

Hence $\varphi(t) = A/\Gamma - (A/\Gamma - \mu) \cdot \exp[-\Gamma \cdot (t - t_0)]$, $t \in (t_0, t_1)$. (5)

Equation (5) can be extended to $t \in (t_0 - \tau_{\max}, t_0)$ during which $u(t) \equiv 1$ and hence the governing equation for $\varphi(t)$ is exactly the same with the assumption $\tau_{max} << t_0$. Also, $\varphi(t - \tau_{\max}) \leq \varphi(t - \tau(t)) \leq \varphi(t - \tau_{\min})$ holds for any $t \in (t_0 - \tau_{\max}, t_0)$ due to the finite bounds of $\tau(t)$ and the monotonically increasing property of $\varphi(t)$.

Define two (bounds) trajectories: $\check{q}_l(t)$, $\hat{q}_l(t)$ at $t \in (t_0, t_1)$ that satisfy equation (4) with fixed delay τ_{max} or τ_{min}, respectively, with the initial state $\check{q}_l(t_0) = \hat{q}_l(t_0) = q(t_0) = 0$. So, we have $\check{q}_l(t) \leq q(t) \leq \hat{q}_l(t)$ for $t \in (t_0, t_1)$. The two trajectories $\check{q}_l(t)$, $\hat{q}_l(t)$ are then called left-sided lower and upper bound curves, respectively.

We can then obtain the solutions of $\breve{q}_l(t)$, $\hat{q}_l(t)$, $t \in (t_0, t_1)$ from equation (5):

$$\begin{cases} \breve{q}_l(t) = (A/\Gamma - \mu)\cdot\left[(t-t_0) - 1/\Gamma\cdot\exp(\Gamma\cdot\tau_{\max})\cdot(1-\exp(-\Gamma\cdot(t-t_0)))\right] \\ \hat{q}_l(t) = (A/\Gamma - \mu)\cdot\left[(t-t_0) - 1/\Gamma\cdot\exp(\Gamma\cdot\tau_{\min})\cdot(1-\exp(-\Gamma\cdot(t-t_0)))\right] \end{cases}. \quad (6)$$

We can assume $\varphi(t_0 - \tau(t_0)) > 0$, by satisfying $\exp(-\Gamma\cdot\tau_{\max}) > 1 - \mu\cdot\Gamma/A$ through parameter design.

The approximations of $(t_1 - t_0)$, denoted as $(\hat{t}_1 - \hat{t}_0)$, $(\breve{t}_1 - \breve{t}_0)$, can be obtained from equation (6) by setting $\hat{q}(t_1) = q(t_1)$ and $\breve{q}(t_1) = q(t_1)$, respectively. It can be shown that $0 \le \hat{t}_1 - \hat{t}_0 \le t_1 - t_0 \le \breve{t}_1 - \breve{t}_0$. Note that $\hat{t}_0 = t_0 = \breve{t}_0$ holds from the definition of the bound trajectories.

With detailed analysis, we can bound $(\breve{t}_1 - \breve{t}_0)$ and $(\hat{t}_1 - \hat{t}_0)$ from (6). As a summary, $(t_1 - t_0)$ is bounded via:

$$\max\left(\tau_{\min},\ \frac{\exp(\Gamma\cdot\tau_{\min})}{\Gamma}\cdot\left[1-\exp\left(\frac{-\Gamma\cdot q(t_1)}{A/\Gamma-\mu}\right)\right]\right) < t_1 - t_0 - \frac{q(t_1)}{A/\Gamma-\mu} < \frac{\exp(\Gamma\cdot\tau_{\max})}{\Gamma} \quad (7)$$

To move any further from (7), we need to study the range of $q(t_1)$. From the definitions of t_0 and t_1, it can be derived from the monotonically increasing property of $\varphi(t)$ in Phase 1 that

$$Q_T \le q(t_1) \le Q_T + (A/\Gamma - \mu)\cdot\tau(t_1) \le Q_T + (A/\Gamma-\mu)\cdot\tau_{\max}. \quad (8)$$

Substituting (8) into (7), we have the following bounds for $(t_1 - t_0)$:

$$\max\left(\tau_{\min},\ \frac{1}{\Gamma}\cdot e^{\Gamma\cdot\tau_{\min}}\cdot\left[1-\exp\left(-\Gamma\cdot\frac{Q_T}{A/\Gamma-\mu}\right)\right]\right) < t_1 - t_0 - \frac{Q_T}{A/\Gamma-\mu} < \tau_{\max} + \frac{1}{\Gamma}\cdot e^{\Gamma\cdot\tau_{\max}}. \quad (9)$$

It is worth noting that alternative bound trajectories can be obtained by evaluating system behavior in Phase 1 from the right side at time t_1. Two trajectories can be defined from the right side at $t \in (t_0, t_1)$: $\breve{q}_r(t)$, $\hat{q}_r(t)$ with fixed delay τ_{min} or τ_{max}, respectively, with the same initial state as $q(t)$ at time t_1. Similarly approximations for $(t_1 - t_0)$ can be obtained (skipped here). Furthermore, the combination of $\breve{q}_l(t)$, $\hat{q}_l(t)$, $\breve{q}_r(t)$, $\hat{q}_r(t)$ as follows can lead to tighter (two-sided) bound trajectories: $\breve{q}(t) \equiv \max[\breve{q}_l(t), \breve{q}_r(t)]$, $\hat{q}(t) \equiv \min[\hat{q}_l(t), \hat{q}_r(t)]$, $t \in (t_0, t_1)$.

The analysis for the other phases are similar but more tedious. So we only present their definitions and then the final results for the single-flow system. Phase 2: $t \in (t_1, t_2)$, where $t_2 \equiv \inf\{t \ge t_1 : q(t - \tau(t)) = Q_T, \varphi(t) < \mu\}$. Phase 3: $t \in (t_2, t_3)$, where $t_3 \equiv \inf\{t \ge t_2 : q(t) = 0, \varphi(t) < \mu\}$. Phase 4: $t \in (t_3, t_4)$, where $t_4 \equiv \inf\{t \ge t_3 : \varphi(t) = \mu\}$. Phase 1–4 together form a "cycle" of the fluctuating solutions in the system working in the fluctuation region. Define the period of an individual fluctuation "cycle" as T, $T \equiv t_4 - t_0$, we have

$$\frac{1}{\Gamma}\cdot e^{\Gamma\cdot\tau_{\min}}\left[1-\exp\left(\frac{-\Gamma\cdot Q_T}{A/\Gamma-\mu}\right)\right] + \tau_{\max} \le T - \frac{Q_T}{A/\Gamma-\mu} \le \frac{1}{\Gamma}\cdot e^{\Gamma\cdot\tau_{\max}} + \frac{2A/\Gamma\cdot(2\tau_{\max}+1/\Gamma)}{A/\Gamma+\mu} + \frac{1}{\Gamma}\ln\left(\frac{A/\Gamma}{A/\Gamma-\mu}\right). \quad (10)$$

Remark 3: for the system working in the fluctuation region:

1 Although the fluctuation is aperiodic, the time duration of each "cycle" is bounded by equation (10). In fact, the time duration of each phase is bounded by equations (9) and the other equations (not listed here).

2 A larger $\tau_{\max}$ tends to increase the length of each "cycle period" and its variation. This makes sense since it is more difficult for the remote user(s) to track the dynamic system behavior with likely larger delays.

3 The parameter design can significantly affect the system behavior. The time duration of each "cycle" largely depends on the ratio of A/Γ and its difference from μ.

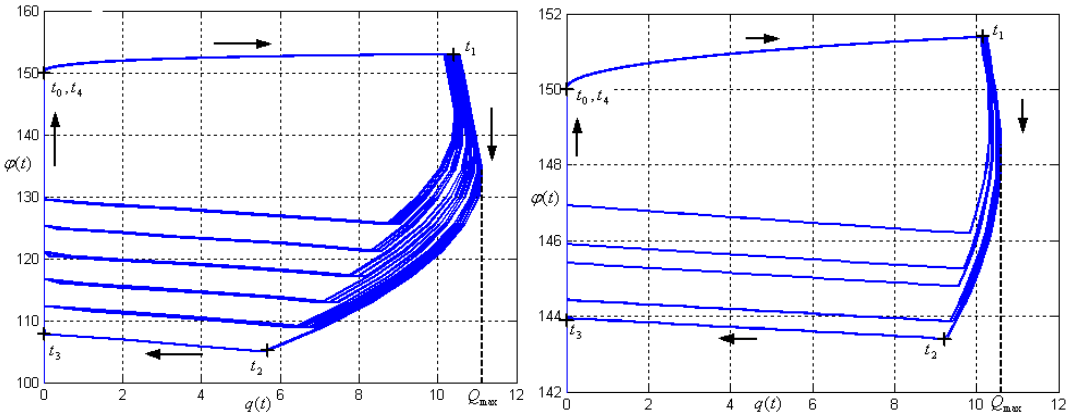
4 The system with larger delays has longer phases in each "cycle", higher overshoot and larger amplitude.

5 With a fixed ratio of A/Γ, a larger Γ speeds up dynamic behavior and in some sense compensate the effects of incorrect feedback due to delays. It results in a higher overshoot and larger amplitude; while a smaller Γ leads to slower system behavior with a lower overshoot. The system with a Γ too small, however, may stay in Phase 4 (and the 2nd half of Phase 3) for a long time, during which the system performance likely degrades.

6 The system with a larger delay or Γ tends to have higher $Q_{\max}$. $Q_{\max}$ is bounded unless $(A/\Gamma - \mu) \to 0^+$.

Figure 2 depicts the "cycles" in the dynamic behavior with fluctuations as an example. In Figure 2(a), the parameters and the time duration are the same as those in Figure 1. The 41 different "cycles" fall into 6 groups; each group has a set of "cycles" whose trajectories are very close with each other. The rate $\varphi(t)$ has a maximum, minimum and average value as 153.0, 105.0 and 144.2 Mbps, respectively. The maximum queue size is 11.1 at φ

= 134.3Mbps. The average T, period of "cycles", is about 12.1s. Its response time, the time duration from 0 to the time when the flow receives the feedback of the remote queue overflow for the first time, is 9.8s.



(a) $A^+ = A^- = 48.1$Mbps, $\Gamma^+ = \Gamma^- = 0.39$ (b) $A^+ = A^- = 4.81$Mbps, $\Gamma^+ = \Gamma^- = 0.039$

$\mu = 150$Mbps, $v = 30$Mbps, $\Delta = 0.05$s, $Q_T = 10$ packets, $\tau(t) = 0.1 + 0.1 \cdot \sin(\pi t/11)$.

Figure 2: Bounded "cycles" in the fluctuation region of single-flow systems

Figure 2(b) shows the same system with the same ratio of A/Γ but different A and Γ as those in Figure 2(a). Compared with Figure 2(a), the time duration of each phase and each "cycle" is longer, so there are less number of "cycles" (about 10) which fall into 5 groups according to their trajectories. Compared with Figure 2(a), the system has a lower Φ_{max} (151.4Mbps), a much higher Φ_{min} (143.4Mbps), a smaller Q_{max} (10.6), a longer T (40.2s) and a higher throughput (148.7Mbps) with much longer response time (98.2s). Furthermore, the rate amplitude decreases dramatically, and hence the relative amplitude, $\Delta\Phi/\overline{\Phi}$, drops from 0.33 to 0.05 accordingly.

3 Models for a multiple flow system

We extend the single-flow model (2) to the multiple-flows model as follows:

$$\frac{d}{dt}\varphi_j(t) = \begin{cases} -\Gamma_j^+[\varphi_j(t) - v_j] + A_j^+ u_j(t) & \text{if} \quad u_j(t) > 0 \\ -\Gamma_j^-[\varphi_j(t) - v_j] + A_j^- u_j(t) & \text{if} \quad u_j(t) < 0 \end{cases} \qquad \text{(a)}$$

$$\frac{d}{dt}q(t) = \begin{cases} \sum_j \varphi_j\left(t - \tau_j(t)\right) - \mu, & \text{if } q(t) > 0 \\ \left(\sum_j \varphi_j\left(t - \tau_j(t)\right) - \mu\right)^+, & \text{if } q(t) = 0 \end{cases}. \qquad \text{(b)} \qquad (11)$$

Here $\Gamma_j^+, \Gamma_j^-, A_j^+, A_j^-$ are nonnegative parameters associated with the j^{th} flow, $j = 1, 2, \ldots, N$, and the feedback for the j^{th} flow at time t is $u_j(t) = \text{sgn}[Q_T - q(t - \tau_j(t))]$. We start from the analysis of the multiple flow systems with stationary solutions. The system that satisfies this condition is stated as the system in the stationary-state region.

3.1 Solutions in the stationary-state region

We have the following results regarding the existence of stationary solution and the stability:

Proposition 3: Suppose $\sum_j \left(v_j + A_j^+/\Gamma_j^+\right) < \mu$. The system in (11) has a stationary solution: $\varphi_j \equiv v_j + A_j^+/\Gamma_j^+$, $\forall j = 1, \ldots, N$ and $q \equiv 0$. Furthermore:

3.1 For a given t_1, if $\varphi_j(t_1) \le v_j + A_j^+/\Gamma_j^+$ $(j = 1, \ldots, N)$, the same inequality also holds for all flows when $t \ge t_1$. If for a flow k at time t_0, $\varphi_k(t_0) > v_k + A_k^+/\Gamma_k^+$, there exists t_2, $t_2 > t_0$, s.t. $\varphi_k(t)$ decreases monotonically when $t_2 > t > t_0$, and $\varphi_k(t_2) = v_k + A_k^+/\Gamma_k^+$.

3.2 For a flow k, assume $\tau_k(t) \in S_T^k \subset R$ and S_T^k is compact. Denote its bounds by $\tau_{\min}^k$ and $\tau_{\max}^k$. Then there exists t_3, s.t. for all $t \ge t_3$, $q(t) < Q_T$, $\varphi_k(t) \to v_k + A_k^+/\Gamma_k^+$ at the exponential rate Γ^+.

We have the similar remarks for the multiple flow system as Remark 1 and Remark 2. The property similarly to equation (3) also holds for each individual flow. Due to space limitation, they are skipped here.

3.2 Solutions in the fluctuation region

In this section we will study the fluctuating but bounded dynamic behavior in the fluctuation region where $\sum_j \left(\nu_j + A_j^+/\Gamma_j^+\right) > \mu$. The assumptions in this section are summarized below ($\forall j = 1, \ ..., \ N$):

- $A_j^+, A_j^- \geq 0$ and $\Gamma_j^+, \Gamma_j^- > 0$. $\Gamma_j^+ = \Gamma_j^- = \Gamma_j$ and $A_j^+ = A_j^- = A_j$, i.e., consider the MS algorithm.
- $\nu_j = 0$, since ν_j can be absorbed by $\varphi_j(t)$ and μ in equation (11).
- $\varphi_j(t)$ and $q(t)$ are piece-wise differentiable functions with $\varphi_j(0) = q(0) = 0$.
- $\tau_j(t) \in S_T \subset R$, where S_T is a compact set with bounds $\tau_j^{\min}$, $\tau_j^{\max}$. Denote $\tau_{\min} \equiv \min_j \tau_j^{\min}$, $\tau_{\max} \equiv \max_j \tau_j^{\max}$.

With the above assumptions, the multiple-flows model (11) can be rewritten as

$$\frac{d}{dt}\varphi_j(t) = -\Gamma_j \cdot \varphi_j(t) + A_j \cdot u_j(t) \qquad \text{(a)}$$

$$\frac{d}{dt}q(t) = \begin{cases} \sum_j \varphi_j\left(t - \tau_j(t)\right) - \mu, & \text{if } q(t) > 0 \\ \left(\sum_j \varphi_j\left(t - \tau_j(t)\right) - \mu\right)^+, & \text{if } q(t) = 0 \end{cases}. \qquad \text{(b)} \qquad (12)$$

It is worth noting that different flows have different $u_j(t)$, and that all flows are coupled in equation (12b). Using a similar but more complicated way as that in the analysis of the single-flow system, we have studied the dynamic behavior of the multiple-flow system with general time-varying delays in the fluctuation region.

Let t_0 be the time when the queue starts buffering the incoming packets for the first time in Phase I. Clearly $q(t_0) = 0$, $0 \leq t_0 < t_{\min}^1$. Let t_j^1 be the time when the j^{th} user receives the feedback information of the remote queue overflow, i.e., $t_j^1 \equiv \inf\left\{ t > 0 : q(t - \tau_j(t)) = Q_T \right\}$. t_j^1 is also called the responsive time of the j^{th} flow. Denote the bounds of the time sequence $\left\{ t_j^1 \right\}_{j=1}^N$ as $t_{\min}^1 \equiv \min_j t_j^1$ and $t_{\max}^1 \equiv \max_j t_j^1$.

We also address the topic of **fairness** with the following constraint among flows:

1) For any flow j, there exists time $t_j \geq 0$, such that $\varphi_j(t) \geq \nu_j$ when $t \geq t_j$.
2) For any two flows j and k, $\left(\varphi_j(t) - \nu_j\right)/\sigma_j = \left(\varphi_k(t) - \nu_k\right)/\sigma_k$ when $t \geq \max(t_j, \ t_k)$.

For the above fairness criteria we consider the following parameter design:

$$\forall j = 1, \ 2, \ ..., \ N, \ A_j = A \cdot \sigma_j, \ \Gamma_j = \Gamma, \text{ and define } \sigma \equiv \sum_j \sigma_j. \qquad (13)$$

With the above design rule, the condition of fluctuation region can be rewritten as $A/\Gamma > \mu/\sigma$. It has been shown in [1] that for the system with heterogeneous fixed delays, this design rule achieves "pointwise fairness": the divergences from the proportional fairness vanish monotonically as $t \to \infty$. Through our analysis, we have:

Proposition 4: Suppose $A/\Gamma > \mu/\sigma$, and the design rule (13) is adopted. $j = 1, \ 2, \ ..., \ N$. So

4.1 $\tau_{\min} \leq t_0 - \frac{1}{\Gamma} \cdot \ln\left[\frac{A/\Gamma}{\left(A/\Gamma - \mu/\sigma\right)} \right] \leq \tau_{\max}$.

4.2 $t_0 \geq \mu/\left(A \cdot \sigma\right) + \sum_j \left(\tau_j^{\min} \cdot \sigma_j/\sigma\right)$.

4.3 $\frac{1}{\Gamma} \cdot \ln\left[\sum_j \sigma_j/\sigma \cdot \exp(\Gamma \cdot \tau_j^{\min}) \right] \leq t_0 - \frac{1}{\Gamma} \cdot \ln\left[\frac{A/\Gamma}{\left(A/\Gamma - \mu/\sigma\right)} \right] \leq \frac{1}{\Gamma} \cdot \ln\left[\sum_j \sigma_j/\sigma \cdot \exp(\Gamma \cdot \tau_j^{\max}) \right]$.

4.4 $t_{\min}^1 - t_0 \geq \tau_{\min} + Q_T/\left(\sigma \cdot A/\Gamma - \mu\right)$.

4.5 $t_{\max}^1 - t_0 \leq \tau_{\max} + \dfrac{Q_T}{A/\Gamma \cdot \sum_j \left(\sigma_j \cdot \left(1 - \exp(\Gamma \cdot \tau_j^{\max})\right)\right) - \mu}$, if $A/\Gamma \cdot \sum_j \left[\sigma_j \cdot \left(1 - \exp(\Gamma \cdot \tau_j^{\max})\right)\right] > \mu$ holds.

4.6 $t_{\max}^1 - t_0 \leq \tau_{\max} + \left(Q_T + A/\Gamma \cdot 1/\Gamma \cdot \sum_j \left(\sigma_j \cdot \exp(\Gamma \cdot \tau_j^{\max})\right)\right)/\left(\sigma \cdot A/\Gamma - \mu\right)$.

4.7 $q(t_j^1) \leq Q_T + \tau_j^{\max} \cdot \left(\sigma \cdot A/\Gamma - \mu\right) \leq Q_T + \tau_{\max} \cdot \left(\sigma \cdot A/\Gamma - \mu\right)$.

Proposition 4 gives the bounds of the sequence of responsive times $\left\{ t_j^1 \right\}_{j=1}^N$ for the rate-control systems in the fluctuation region. The difference between A/Γ and μ/σ is one of the most important factors in the bounds: a smaller difference leads to much longer responsive times. In general, for a specific flow j, the bounds of its responsive time t_j^1 increase with the feedback delay bounds of the j^{th} flow $\tau_j^{\min}, \tau_j^{\max}$. Because $\left\{ t_j^1 \right\}_{j=1}^N$ are delayed tracking of a common queue in different flows, the flow with less delay always has the shorter responsive time. As we all know, the (local) maximum rate $\varphi_j^{\max}$ increases with the corresponding responsive time t_j^1.

In our parameters design, when A/Γ is fixed, to compensate for the long responsive time, the system may need a large Γ or a large σ (or both). In addition, for the same purpose, we can adjust the relative weights σ_j, $j = 1, 2, ..., N$ by assigning smaller relative weights to the flows with longer delays.

Proposition 4.7 presents an upper bound of the queue size at the responsive time of each flow, which can be used along with the governing equation (12) to bound the (local) maximum queue size $q_{\max}$:

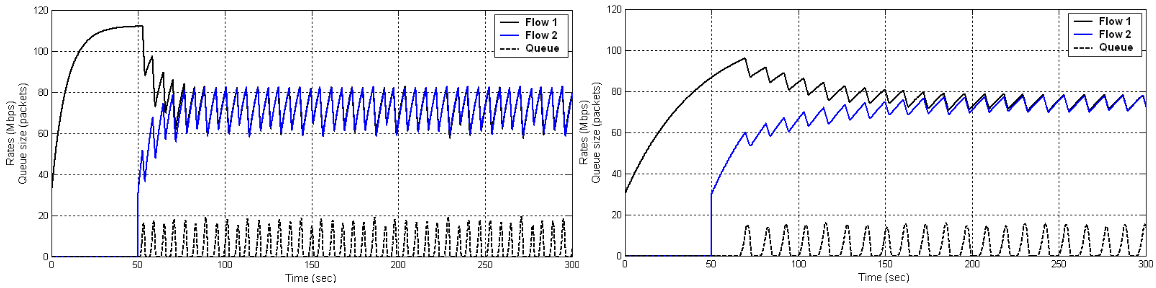
$$q_{\max} \le \max_j \left[2 \cdot q(t_j^1) - Q_T\right] \le \max_j \left[Q_T + 2 \cdot \tau_j^{\max} \cdot (\sigma \cdot A/\Gamma - \mu)\right] \le Q_T + 2 \cdot \tau_{\max} \cdot (\sigma \cdot A/\Gamma - \mu) \qquad (14)$$

In summary, a larger A/Γ (or σ) leads to shorter responsive times and lower maximum rates, with the tradeoff of a larger queue size. The tradeoffs in the parameters design are necessary to give consideration to both issues to achieve small responsive times of all flows and less overshoots if any with a reasonable queue size.

4 Simulation results of the system with multiple flows

Extensive simulation have been done for the multi-flow system with heterogeneous time-varying feedback delays, to demonstrate the issues, qualitatively and quantitatively, such as transient behavior, parameter design, the effect of time-varying delays, fairness, etc. Due to space limitation, only selective simulation are shown here.

Figure 3 shows two flows with the same weight but different time-varying delays starting at time 0 and 50s, respectively. $\tau_1^{\min} = \tau_2^{\min} = 0.05$s, $\tau_1^{\max} =$ 0.15s and $\tau_2^{\max} = 0.25$s. For convenience the queue size is measured in units of nominal packets. 1 nominal packet has 6250 bytes, i.e., the product of 1Mbps/s and 0.05s.



(a) A = 9.62Mbps, Γ = 0.117 (b) A = 1.924Mbps, Γ = 0.0234

Flow 1 and 2 starts at 0 and 50s, respectively. μ = 150Mbps, $\nu_1 = \nu_2$ = 30Mbps, Δ = 0.05s, Q_T = 10 packets, $\sigma_1 = \sigma_2 = 1$, $\tau_1(t) = 0.1 + 0.05 \cdot \sin(\pi t/11)$, $\tau_2(t) = 0.15 + 0.1 \cdot \sin(\pi t/7)$.

Figure 3: Multi-flows: various gain and damping constants with fixed ratio

In Figure 3, for t < 50s the system has only flow 1, and $\sigma_1 \cdot A/\Gamma + \nu_1 < \mu$, so the system has stationary-state solutions. The rate of flow 1 approaches the steady state exponentially and the queue is always empty. For $t \ge$ 50s, flow 2 attempts to obtain its share of the bandwidth, and $\sum_{j=1,2} \left(\sigma_j \cdot A/\Gamma + \nu_j\right) > \mu$. So the system is in the fluctuation region for $t \ge$ 50s; the fluctuating behavior of the rates and queue size can be clearly observed. With different delays, two flows have different aperiodic fluctuations. However, the amplitude and period of the fluctuations are bounded and close to periodic; and the rates of the two flows almost coincide with each other after the transient period. It follows that fairness (½ to ½) is achieved between two flows at almost any time.

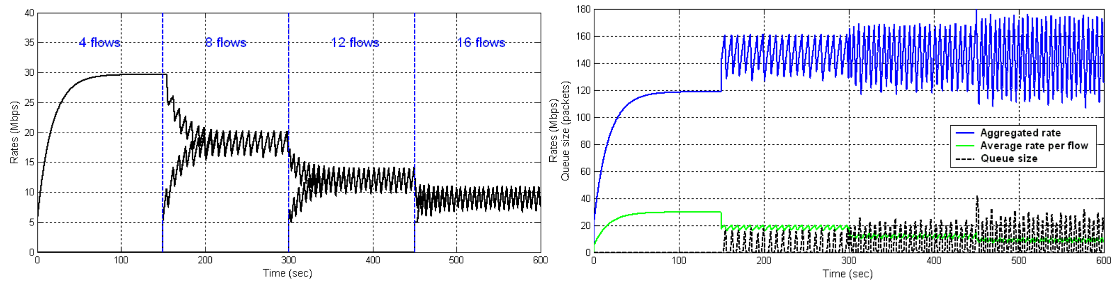
Two sets of gains and damping constants are used in Figure 3 with their ratio fixed: the one in (a) has larger (A, Γ), which leads to shorter responsive times in both stable and fluctuation regions; while the other set in (b) provides smaller fluctuations and requires less buffer size in the common queue. Either of them may be desirable in practice depending on the specific purpose of parameter design; or it could be any other set in between for further tradeoff among the above performance metrics.

Figure 4 shows the transient behavior of the system with 16 flows as four new flows start at 0, 150s, 300s and 450s, respectively. The rates of all flows are shown in Figure 4(a) while the aggregated rate, the average rate per flow and the queue size are shown in Figure 4(b). For t < 150s, the system has stationary solutions with an aggregated rate of 120Mbps; for $t \ge$ 150s, the system has fluctuating solutions. Note that the rate fluctuations slow down as the number of flows increases. Figure 4(c) shows the Jain's Fairness Index (*FI*) at different times with 4, 8, 12 and 16 active flows, respectively. The minimum bandwidth is not considered in the allocated resource, i.e., $x_i(t) = \varphi_i(t) - \nu_i$, where flow i is an active flow. The perfect fairness (*FI* ≈ 1) can be clearly observed except for the transient times when a new group of flows have been just started.

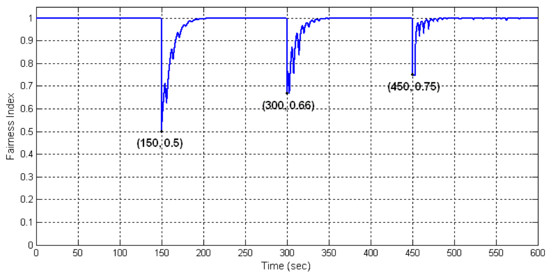
5 Conclusions

In this paper, we focus on feedback-based rate control systems for adaptive bandwidth allocation in broadband satellite communication networks. Our analyses are based on analytic fluid models composed of first-order delay-differential equations with damping and gain functions. Furthermore, practically and most importantly, the heterogeneous time-varying propagation delays are reflected in the system models. Single-flow and multi-flow system models are analyzed, respectively, with much attention paid to the symmetrical Mitra-Seery (MS) algorithm.

We show the stationary solutions, existence conditions and convergence speed for the single-flow and multi-flow system models, respectively. And then for the situations under which the systems only have fluctuating solutions, we analyze the dynamic behavior of rates and queue size in detail. Based on the analytic results, we investigate the effect of delays and parameters in terms of fairness, fluctuation (amplitude, period), transient behavior and adaptability, etc. It has been shown, analytically and in simulations, that with proper parameter design the system can achieve stable behavior with close to pointwise proportional fairness among flows.



(a) Rates of flows (b) Aggregated rate and queue behavior



Four new flows start at 0, 150, 300 and 450 s, respectively. A = 4.81Mbps, Γ = 0.0585, μ = 150Mbps, Δ = 0.05s, Q_T = 10 packets, v_j = 5Mbps, σ_j = 0.3, j = 1, …, 16.
For k = 0, 1, 2, 3,
$\tau_{4k+1}(t) = \tau_{4k+4}(t) = 0.1 + 0.05 \cdot \sin(\pi t/11)$,
$\tau_{4k+2}(t) = \tau_{4k+3}(t) = 0.15 + 0.1 \cdot \sin(\pi t/7)$.

Figure 4: Multi-flows: the effect of increasing number of flows

References

1 F. Bonomi, et al, “Adaptive algorithms for feedback-based flow control in high-speed wide-area ATM networks”, IEEE JSAC, vol. 13 (1995), issue 7, pp. 1267–1283.
2 A. I. Elwalid, “Adaptive rate-based congestion control for high speed wide-area networks: stability and optimal design”, Proc. of ICC 1995, pp. 1948–1953, San Francisco, CA.
3 K. K. Ramakrishnan, et al, “A binary feedback scheme for congestion avoidance in computer networks with a connectionless network layer”, ACM SIGCOMM ’88, pp. 303–313.
4 S. Keshav, “Packet-Pair Flow Control”, IEEE/ACM Transactions on Networking, February 1995.
5 R. J. La and P. Ranjan, “Stability of rate control system with time-varying communication delays”, Technical Report, ISR TR 2004-31, 2004.
6 F. Mazenc, et al, “Remarks on the stability of a class of TCP-like congestion control models”, Proc. 42nd IEEE Conference on Decision and Control, Vol. 6, 9-12, pp. 5591 – 5594, Dec. 2003.
7 T. Alpcan and T. Basar, “A utility-based congestion control scheme for Internet-style networks with delay”, INFOCOM 2003.
8 R. Johari and D. Tan, “End-to-end congestion control for the Internet: delays and stability”, IEEE/ACM Transactions on Networking, Vol. 9, Issue 6, pp. 818-832, Dec. 2001.
9 S. Deb and R. Srikant, “Global stability of congestion controllers for the Internet”, in Proc. of the 41st IEEE Conference on Decision and Control, Vol. 4, 10-13, pp. 3626-3631, Dec. 2002.
10 P. Ranjan, R. J. La, and E. H. Abed, “Global stability in the presence of distributed communication delays”, American Control Conference 2005, Portland, OR, 2005.
11 H. Chen, Ph.D. dissertation, “Network flow optimizations and distributed control algorithms”, University of Maryland, College Park, MD, 2006.
12 K. W. Fendick, M. A. Rodrigues and A. Weiss, “Analysis of rate-based feedback control strategy for long haul data transport”, Performance Evaluation, vol. 16, pp. 67-84, 1992.