Aerial Platform Placement Algorithm to Satisfy Connectivity and Capacity Constraints in Wireless Ad-hoc Networks

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***Abstract*—In this paper, we address the problem of establishing full connectivity and satisfying required traffic capacity between disconnected clusters in large wireless ad-hoc ground networks by placing a minimum number of advantaged high flying Aerial Platforms (APs) as relay nodes at appropriate places. We formulate the problem of providing both connectivity and required capacity between disconnected ground clusters as a constrained clustering problem with complexity costs. The basic requirement for connectivity between the ground clusters and APs is converted into a summation form distortion function. The additional requirement for connectivity between the various APs is encoded by adding a new (summation form) constraint to the distortion function. In order to satisfy the required capacity out of each cluster to all other clusters, we add a cost function that depends on the assignment probabilities of the APs and relate the source (prior) probabilities of each cluster to the required capacity out of this cluster. The cost function produces solutions which are load balanced, i.e., the capacities supported through each AP are nearly equal. We solve the resultant clustering problem using Deterministic Annealing in order to find (near) globally optimal solutions for the minimum number and locations of the APs to establish full connectivity and provide required traffic capacity between disconnected clusters. We establish the validity of our algorithm by comparing it with optimal exhaustive search algorithms and show that our algorithm is near-optimal for the problem of establishing connectivity and satisfying capacity requirements between disconnected clusters.**

I. Introduction

Wireless Mobile Ad Hoc Networks (MANETs), where nodes form and maintain a wireless multihop network without any central infrastructure, are becoming popular both in the commercial and the military world. It is highly probable that a MANET has nodes that are disconnected from each other. Also in the case of military or disaster relief networks, the operational scenario may be such that there are disconnected clusters of nodes but still there is need for communication between the different clusters. One of the methods suggested to improve connectivity, capacity, robustness, and survivability of MANETs is to use Aerial Platforms (APs) as relays in the network.

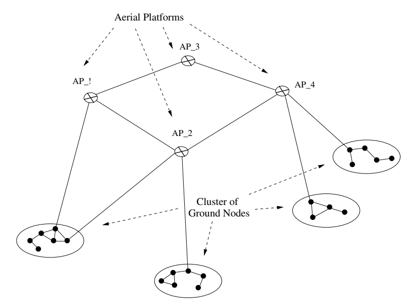

Fig. 1: Network with four partitions and four connecting APs.

In this paper, we look at the problem of providing connectivity between disconnected ground clusters and satisfying required traffic capacity between these clusters by placing a number of APs at appropriate places to act as relay nodes. Since Aerial Platforms are scarce and expensive resources, the goal is to find the minimum number of APs and their locations so that the resultant network (both between the ground nodes and the APs and between the APs) is connected and there are enough pathways to support the required inter-cluster capacity (see Figure 1). In [1], the authors use a deterministic annealing (DA) clustering approach ([2]) to find near-optimal solutions to the problem of finding the minimum number of APs and their location so that at least one node from each ground cluster is connected to at least one AP. We extend the approach of [1] in two ways: *a*) include communication distance constraints between the APs so that not only are the clusters connected to the APs but the APs also form a connected network; *b*) the APs connected to each cluster are capable of supporting the required capacity out of each cluster to other clusters with maximum AP-cluster link utilization.

The paper is organized as follows. Section II describes our scenario and the assumptions made. Section III explains our formulation of the connectivity problem and the capacity constrained problem in the framework of a constrained clus-

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This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2008 proceedings.

tering problem with complexity costs. Section IV explains the DA solution to the problem and gives a brief review of the algorithm used. Section V presents the results of the algorithm where we also compare our results with an exhaustive grid search algorithm. Finally, we conclude in section VI.

II. Scenario and Assumptions

Let the ground nodes and the APs have identical omni-directional radios with free space communication (where the signal decays as $1/R^2$, with R being the distance between radios) possible if the distance between two radios is less than R_2. Since the ground nodes communicate with one another in an environment (indirect reflections, etc.) where the signal decays as $1/R^\alpha$, where α is greater than 2 (suburban decay is as $1/R^4$), we assume that the ground nodes can communicate with each other if their distance is less that R_0 (with $R_0 < R_2$). Assume that the ground network has N nodes (with positions $G = \{g_i,\ i = 1, \ldots, N\}$) forming M clusters where the nodes within each cluster can communicate with each other and the nodes in different clusters cannot communicate with one another. Each cluster is represented by K_j, $j = 1$ to M. Also assume that all of the ground nodes, g_i ($i = 1, \ldots, N$), have the same altitude (of 0). This assumption basically keeps the problem in $\mathcal{R}^2$ and is a reasonable approximation for most practical cases.

Let each AP fly at a maximum cruising altitude of h in a holding pattern above the scenario. Since the AP-AP and AP-ground node communication can be modeled as that of free space, it is assumed that the AP-AP or AP-ground node communication can take place if the distance between the nodes is less than R_2. Since all APs fly at a constant altitude h, the connectivity problem can be reduced to $\mathcal{R}^2$, with the positions of the APs projected onto the ground and denoted by a_k (with $A = \{a_k, k = 1, \ldots, L\}$ assuming L APs). This results in a maximum AP-ground node communication distance of $R_1 = \sqrt{R_2^2 - h^2}$ with the AP-AP maximum communication distance being R_2.

Assume that the maximum link capacity between AP-AP and AP-ground node is C_{max}. Let the total capacity required from source cluster K_i to destination cluster K_j be C_{ij} with C_{ii} taken to be 0. Hence the total capacity (C_i) of the links going out and coming into cluster K_i is $C_i = \sum_{j=1}^{K}(C_{ij} + C_{ji})$. Since the maximum AP-ground node capacity is C_{max}, we need to have $C_i \leq C_{max} (\forall i = 1, \ldots, M)$.

III. Formulation of the Problem

If the baseline ground scenario is disconnected, Aerial Platforms can be used to establish connectivity and provide required capacity. We formulate the connectivity problem as a constrained clustering problem ([3], [4]) with a summation form distortion function ($D(K, A)$) involving the distances between the ground clusters (K) and the APs (A) and a summation form cost function ($C_1(A)$) involving only the

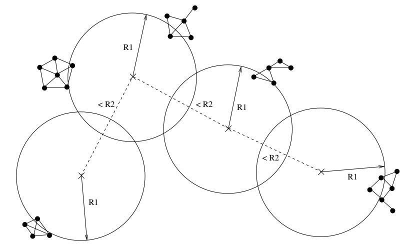

Fig. 2: Aerial Platform Placement.

distances between the APs (A). The capacity constraints, including maximizing the AP-cluster link utilization, are handled by adding a complexity cost function $C_2(p(A))$ ([5]) that only depends on the *assignment probabilities* $p(a_i)$ of the APs; and relating the prior probabilities $p(K_i)$ of each cluster K_i to be proportional to C_i. The resultant clustering problem is then solved using Deterministic Annealing (DA) to obtain near-optimal solutions.

A. Deterministic Annealing

Deterministic Annealing ([2]) is a method for clustering where a large number of data points, denoted by x's, (in our problem, the various ground clusters) need to be assigned to a small number of centers, denoted by y's, (in our problem, the various APs) such that the average distortion function is minimized. The average distortion can be written as $D = \sum_x p(x)d(x, y(x))$, where $p(x)$ is the prior probability of data point x. The DA approach tries to avoid local minima by turning the hard clustering problem (where a data point is associated with only one center) into a soft/fuzzy clustering problem (where each data point can be associated to many centers via its *association probabilities* $p(y|x)$) and then minimizing the distortion at various levels of randomness measured by the Shannon entropy $H(X, Y)$. Hence the original distortion function is re-written as $D = \sum_x p(x) \sum_y p(y|x)d(x, y)$ where the *assignment probability* $p(y) = \sum_x p(x)p(y|x)$, measures the percentage of data points assigned to a center y. The objective function that DA minimizes is $F = D - TH$ or $F = D - TH(Y|X)$ at various values of temperature T starting from high temperature and then slowly decreasing the temperature.

B. Connectivity Problem

In order to connect the various ground clusters to the APs while ensuring that the APs form a connected network, we need to find the minimum number of APs L and their positions on the ground, a_k, (with $A = \{a_k, k = 1, \ldots, L\}$) such that:

- At least one node from each cluster is within a radius of R_1 from an AP (see Figure 2); and
- The AP locations a_k are within R_2 from each other (i.e., the APs form a connected graph; see Figure 2).

Assuming that the APs are numbered from 1 to L, we can make sure that the APs form a connected network by ensuring that any AP numbered j is connected to atleast one lower numbered AP i, where $i < j$. This is used in the DA solution where when we add a new AP, we make sure that it is connected to at least one of the previously added APs. Hence the connectivity problem can be stated as:

$$
\begin{array}{c}
Minimize\ L \\
subject\ \ to \\
\exists a_1,\ldots,a_L; \quad \max_{j\in\{1,\ldots,M\}} \ \min_{\substack{g\in K_j \\ i\in\{1,\ldots,L\}}} \parallel g - a_i \parallel \leq \ R_1 \\
\text{and,} \qquad \max_{l\in 2,\ldots,L} \ \min_{m<l} \parallel a_l - a_m \parallel \leq \ R_2
\end{array}
$$

where $\parallel g - a \parallel$ is the l^2-norm between points g and a on the ground. Finding the exact solution to the problem above involves an exhaustive search on the different ways in which nodes can be selected from each cluster and the ways clusters can be grouped together for coverage by a single AP all the while making sure that the APs are connected to each other. This problem is NP-hard as it is a generalization of the Euclidean *disk-cover* problem [6]. Hence using the approximation,

$$\max(s_1,\ldots,s_n) \cong (s_1^\alpha + \ldots + s_n^\alpha)^{\frac{1}{\alpha}} \ \text{ for large } \alpha$$

we can convert the AP-ground node and AP-AP constraints into a summation form,

$$
\begin{array}{rcl}
 & Minimize\ L & \\
 & subject\ \ to & \\
\exists a_1,\ldots,a_L; & \displaystyle\sum_{j=1}^{M} d_1(K_j, a_{u_1(j)}) & \leq \quad R_1^\alpha \\
\text{and,} & \displaystyle\sum_{l=2}^{L} d_2(a_l, a_{u_2(l)}) & \leq \quad R_2^\beta
\end{array}
$$

for large α and β, where,

$$
\begin{aligned}
d_1(K_j, a_i) &= \min_{g\in K_j} \parallel g - a_i \parallel^\alpha \\
d_2(a_l, a_m) &= \min_{m<l} \parallel a_l - a_m \parallel^\beta \\
u_1(j) &: \{1,\ldots,M\} \rightarrow \{1,\ldots,L\}
\end{aligned}
$$

is the function that assigns an AP to every cluster.

$$u_2(l) : \{2,\ldots,L\} \rightarrow \{1,\ldots,L-1\}$$

is the function that assigns the closest lower numbered AP to an AP.

Within the framework of constrained clustering ([3], [4]), the distortion function between the ground nodes and the APs is given by $D(K,A) = \sum_{j=1}^{M} d_1(K_j, a_{u_1(j)})$ and the cost function among the APs is given by $C_1(A) = \sum_{l=2}^{L} d_2(a_l, a_{u_2(l)})$.

C. Capacity Constraints

In order to ensure that the capacity required by a cluster K_i to communicate with other clusters (i.e., C_i) is satisfied by the APs within communication range of the cluster, we need to ensure that the capacity supported by an AP, ($C_{ap}(j) \triangleq \sum_{i=1}^{M} C_i\, I(\text{AP } j \text{ is associated with cluster } i)$), is less than the maximum link capacity C_{max}. Since in the clustering formulation, we can have a single cluster K_i associated with different APs via its association probabilities $p(a_j|K_i)$, we rewrite the capacity supported by an AP, $C_{ap}(j)$ as

$$C_{ap}(j) = \sum_{i=1}^{M} C_i\, p(a_j|K_i) \leq C_{max}$$

Denoting the AP-cluster link utilization as $u(j) = C_{ap}(j)/C_{max}$, in order to maximize the sum of the AP-cluster link utilizations, we would like to $max \sum_{j=1}^{L} u(j)$.

In order to satisfy the capacity constraints from each cluster K_i to all other clusters, we let the cluster prior probability be set to $p(K_i) = C_i / \sum_{j=1}^{M} C_j$ and add a complexity cost function $C_2(p(a_k)) = 1/p(a_k)^s$ that only depends on the assignment probabilities of the APs. For high values of s, the cost value for small $p(a_k)$ (i.e., $1/p(a_k)^s$) blows up and the end resultant solution ([5]) tends to be load balanced, i.e., $p(a_k) = 1/L$, $\forall k = 1,\ldots,L$. But

$$
\begin{aligned}
p(a_k) &= \sum_{i=1}^{M} p(K_i)\, p(a_k|K_i) \\
&= \sum_{i=1}^{M} \left(C_i / \sum_{j=1}^{M} C_j \right) p(a_k|K_i) \\
\Rightarrow\ p(a_k) \sum_{j=1}^{M} C_j &= \sum_{i=1}^{M} C_i\, p(a_k|K_i) \ = \ C_{ap}(k)
\end{aligned}
$$

Hence we stop adding APs when the maximum of $p(a_k) \sum_{j=1}^{M} C_j$ over all the APs becomes less than the maximum link capacity C_{max}. Since all the $p(a_k)$'s are approximately equal, we also tend to maximize the sum of the AP-cluster link utilizations.

IV. Deterministic Annealing Solution

The overall distortion function D including the AP-AP connectivity constraints and the cluster capacity constraints is given by:

$$
\begin{aligned}
D \ = \ & \sum_{i=1}^{M} p(K_i) \sum_{j=1}^{L} p(a_j|K_i) \left[d_1(K_i, a_j) + \eta\, C_2(p(a_j)) \right] \\
& + \lambda \sum_{l=2}^{L} d_2(a_l, a_{u_2(l)})
\end{aligned}
$$

The deterministic annealing algorithm tries to minimize the objective function $F = D - T\, H(A|K)$ where

$$H(A|K) = -\sum_{i=1}^{M} p(K_i) \sum_{j=1}^{L} p(a_j|K_i) \log p(a_j|K_i).$$

Minimizing F with respect to the association probabilities $p(a_j|K_i)$ with the additional constraints that $p(a_j) =$

$\sum_{i=1}^{M} p(K_i)p(a_j|K_i)$ and $\sum_{j=1}^{L} p(a_j|K_i) = 1$ gives the Gibbs distribution:

$$p(a_j|K_i) = \frac{\exp\left(-\frac{d_1(K_i,a_j)+\eta C_2(p(a_j))+\eta p(a_j)\frac{dC_2(p(a_j))}{dp(a_j)}}{T}\right)}{Z_{K_i}}$$

where

$$Z_{K_i} = \sum_{j=1}^{L} \exp\left(-\frac{1}{T}(d_1(K_i,a_j) + \eta C_2(p(a_j)) + \eta p(a_j)\frac{dC_2(p(a_j))}{dp(a_j)})\right)$$

The corresponding minimum F^* of F is obtained by plugging the values for $p(a_j|K_i)$ into $F = D - T\,H(A|K)$ to obtain:

$$F^* = -T\sum_{i=1}^{M} p(K_i)\log Z_{K_i} - \eta \sum_{j=1}^{L} p^2(a_j)\frac{dC_2(p(a_j))}{dp(a_j)} + \lambda \sum_{l=2}^{L} d_2(a_l, a_{u_2(l)})$$

The optimal AP locations a_k are given by minimizing F^* leading to the following expression involving the gradient of a_k that needs to be set to zero:

$$\sum_{j=1}^{M} p(K_j, a_k)\,\nabla_{a_k}\,(d_1(K_j,a_k)) + \lambda\,\nabla_{a_k}\left(\sum_{l=2}^{L} d_2(a_l, a_{u_2(l)})\right)$$

This leads to two equations, one for the x coordinate of a_k (i.e., x_{a_k}) and another for the y coordinate of a_k (i.e., y_{a_k}):

$$x_{a_k} = \frac{\alpha\sum_{j=1}^{M} d1X_{numr}(K_j,a_k) + \lambda\,\beta\,(d2X_{numr}(a_k))}{\alpha\sum_{j=1}^{M} d1_{denr}(K_j,a_k) + \lambda\,\beta\,(d2_{denr}(a_k))}$$

$$y_{a_k} = \frac{\alpha\sum_{j=1}^{M} d1Y_{numr}(K_j,a_k) + \lambda\,\beta\,(d2Y_{numr}(a_k))}{\alpha\sum_{j=1}^{M} d1_{denr}(K_j,a_k) + \lambda\,\beta\,(d2_{denr}(a_k))}$$

where

$$d1X_{numr}(K_j,a_k) = x_{K_j}\,p(K_j)p(a_k|K_j)d_1(K_j,a_k)^{1-2/\alpha}$$
$$d1Y_{numr}(K_j,a_k) = y_{K_j}\,p(K_j)p(a_k|K_j)d_1(K_j,a_k)^{1-2/\alpha}$$
$$d1_{denr}(K_j,a_k) = p(K_j)p(a_k|K_j)\,d_1(K_j,a_k)^{1-2/\alpha}$$

$$d2X_{numr}(a_k) = x_{a_{u_2(k)}}\,d_2(a_k,a_{u_2(k)})^{1-2/\beta} + \sum_{l>k} I(u_2(l)=k)\,x_{a_l}\,d_2(a_l,a_k)^{1-2/\beta}$$

$$d2Y_{numr}(a_k) = y_{a_{u_2(k)}}\,d_2(a_k,a_{u_2(k)})^{1-2/\beta} + \sum_{l>k} I(u_2(l)=k)\,y_{a_l}\,d_2(a_l,a_k)^{1-2/\beta}$$

$$d2_{denr}(a_k) = d_2(a_k,a_{u_2(k)})^{1-2/\beta} + \sum_{l>k} I(u_2(l)=k)\,d_2(a_l,a_k)^{1-2/\beta}$$

For $k = 1$, $d_2(a_k, a_{u_2(k)}) = 0$, so that the first term in $d2X_{numr}(a_k)$, $d2Y_{numr}(a_k)$, and $d2_{denr}(a_k)$ is not present.

A. Algorithm

We start with an initial temperature $T = T_{init}$ and $\lambda = 0$ to get the unconstrained clustering solution for that T (i.e., taking into account only the connectivity between the APs and ground clusters). At a given T, we then gradually increase λ and optimize until the maximum of the minimum inter-node distance between an AP and its lower numbered APs is just less than R_2. We then reduce the temperature T and repeat the procedure of increasing λ from 0. The temperature T is progressively reduced until all the clusters are covered by at least one AP and the capacity constraints are satisfied, i.e., $\max_{k=1}^{L} p(a_k)\sum_{j=1}^{M} C_j \leq C_{max}$.

At each iteration (i.e., fixed T and fixed λ), the association probabilities $p(a_i|K_j)$ are first calculated, then the assignment probabilities $p(a_i)$ are calculated, and finally the optimal AP locations a_i are determined until there is convergence. If after a fixed number of iterations, either all the clusters are not covered or the cluster capacity constraints are not satisfied, then the number of APs is increased. This is done by choosing the AP center i with either farthest associated groups or maximum $p(a_i)$ and adding a small perturbation to its current location, and then dividing its probability $p(a_i)$ equally between the new and old center. If a new center is really needed, then the two centers move apart from each other, else they merge again after a few steps. This is checked by finding the distance between the new and old centers after a couple of iterations and merging them if the distance is less than a threshold.

V. Results

A. Connectivity Constraints

To test the connectivity solution, we fix the inter-ground node communication distance R_0 to 0.1 and AP-ground node communication distance R_1 to 0.2. We set AP-AP communication distance to $R_2 = 2 * R_1$. In the resulting configuration the APs are not connected. Using the constrained clustering formulation taking into account the inter-AP connectivity, we obtain the output shown in Figure 3. We see that 6 APs are necessary for connecting the APs with one another and ensuring that all clusters are connected to at least one AP.

The results of the constrained clustering formulation with inter AP connectivity are compared with a Grid algorithm that performs an exhaustive search over the ground node area to find the minimum number of APs required to connect the different clusters and also have connectivity among themselves. The Grid algorithm divides the area into a grid with a granularity of 0.02. The algorithm starts with a single AP and then increments the number of APs until a solution is found. Obviously, this procedure is not scalable and can only be used in relatively small scenarios. The Grid algorithm when run with the same 170 node scenario also requires a minimum of 6 APs to ensure full connectivity both among the ground clusters and among each other. The output of the grid algorithm with $R_2 = 0.4$ is shown in figure 4.

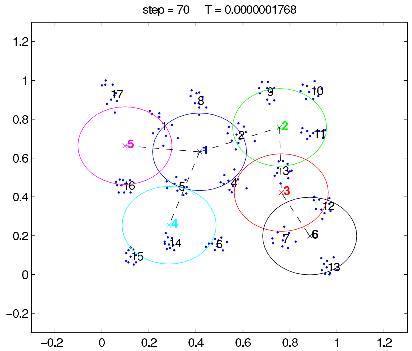

Fig. 3: Complex Scenario: AP Placement with AP-ground node connectivity and AP-AP connectivity.

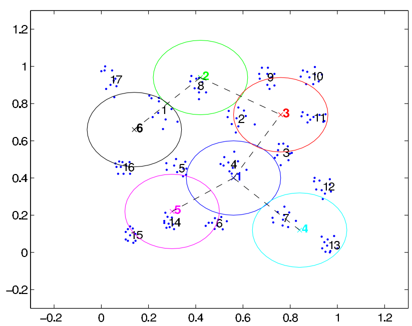

Fig. 4: Grid Algorithm (with $R_2 = 0.4$): Number and location of APs.

B. Capacity Constraints

To test the inclusion of capacity constraints, we used a simple scenario of 4 nodes arranged on the corners of a square with sides 0.35 (see figures 5 and 6). R_0, R_1, and R_2 are the same as in the previous section. C_1 (total capacity out of node 1 to all other nodes) and and C_2 are set to 0.4 Mbps each. The corresponding capacity for nodes 3 and 4 is set to 0.8 Mbps. C_{max} is set to 1.0 Mbps. If capacity constraints are taken into account, a single AP can support both nodes 1 and 2 while nodes 3 and 4 need a separate AP each. Thus the minimum number of APs taking into account capacity constraints is 3 and this is shown in figure 6. The solution without taking into account capacity constraints requires 2 APs for full connectivity as seen in figure 5.

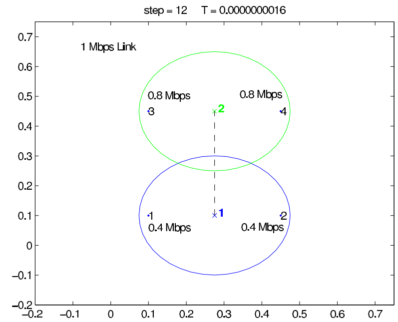

Fig. 5: Simple 4 Node Scenario: AP Placement for full connectivity without capacity constraints.

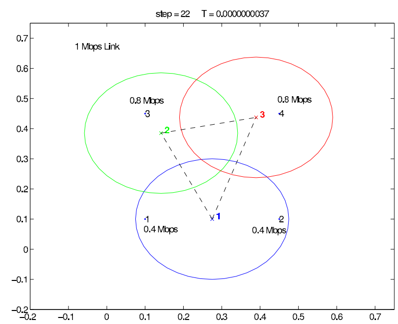

Fig. 6: Simple 4 Node Scenario: AP Placement for full connectivity with capacity constraints.

VI. Conclusions

We have addressed the problem of providing full connectivity between disconnected ground clusters while at the same time satisfying required inter-cluster capacities by placing a minimum number of Aerial Platforms at appropriate locations. This problem is critical in ad hoc networks that need to have full connectivity and enough capacity between all ground nodes like in battlefield networks, rescue scenarios, etc. We use a constrained clustering formulation with complexity costs for solving this problem. The Deterministic Annealing clustering algorithm is used to avoid local minima and obtain near-optimal solutions. Our method for providing full connectivity is validated against an exhaustive search algorithm.

Acknowledgment

This work was supported by SBIR Phase I contract W15P7T-07-C-P402 and Option from U.S. Army CECOM to AIMS, Inc.

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