Coalition Formation through Learning in Autonomic Networks

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Abstract— **Autonomic networks rely on the cooperation of participating nodes for almost all their functions. However, due to resource constraints, nodes are generally selfish and try to maximize their own benefit when participating in the network. Therefore, it is important to study mechanisms, which can be used as incentives for cooperation inside the network. In this paper, the interactions among nodes are modelled as games. A node joins a coalition if it decides to cooperate with at least one node in the coalition. The dynamics of coalition formation proceed via nodes that interact strategically and adapt their behavior to the observed behavior of others. We present conditions that the coalition formed is stable in terms of Nash stability and the core of the coalitional game.**

I. Introduction

Autonomic networks rely on the cooperation of participating nodes for almost all their functions, for instance, to route data between source and destination pairs that are outside each other's communication range. However, because nodes are resource constrained, we deal with networks composed of selfish users who are trying to maximize their own benefit from participation in the network. In particular, we assume that each user is in complete control of his network node. In the routing example, the fundamental user decision is between forwarding or not forwarding data packets sent by other users. Given the constraints (mostly related to battery power) that the user faces, there is a very real cost incurred when choosing to forward. So, all users would like to send their own data packets, but not forward those of other users. Unfortunately, if all users were to do that, the network would collapse. In order to form the necessary infrastructure that makes multi-hop communications achievable, cooperation enforcement mechanisms are needed to cope with such selfish behavior of nodes in autonomic networks.

The conflict between the benefit from cooperation and the required cost for cooperation naturally leads to game-theoretic studies, where each node strategically decides the degree to which it volunteers its resources for the common good of the network. The players in game theory attempt to maximize an objective function which takes the form of a payoff. Srinivasan et al. [1] address the problem of cooperation among energy constrained nodes and devised behavior strategies of nodes that constitute a Nash equilibrium. In [2], there is a link between two nodes if they agree to cooperate. These links are formed through one-to-one bargaining and negotiation.

For any node, the benefit of cooperation comes not just from nodes directly connected (one-hop), but also from nodes that are indirectly connected (multi-hop, through other users). For instance, in multi-hop wireless networks, this is the incentive the users have for forwarding packets. In other words, by activating a communication link towards one of their neighbors, they gain by having access to the users with which that neighbor has activated *his* links, and so on, recursively. The more users a user has access to, the more desirable it is for his neighbors to activate their link towards him.

Therefore, in this paper, we study cooperation and games based on the notion of *coalitions*. The concept of users being connected to each other, and – by getting connected – acquiring access to all the other users that each of them had so far access to, can be well captured by cooperative game theory (also known as coalitional game theory [3]). A question that has only relatively recently began to attract attention ([4] is the first work in this area) is the actual way the coalition is formed. There has been extensive research on coalition formation in the context of social and economic networks [5], [6]. The cooperative game is usually modelled as a *two-period structure*. Players must first decide whether or not to join a coalition. This is done by pairwise bargaining, in which *both* players have to agree to join in a coalition. In the second step, players in the coalition negotiate the payoff allocation. The central problem is to study the payoff allocation scheme and whether the scheme results in a stable solution. In our previous work [7], we studied such two-phase games in the context of communication networks and investigated the fundamental tradeoffs between the gain and cost of collaboration.

In this paper, we study another type of iterated games, where the payoff of players depends on the coalition structure they belong to, and where the payoff changes with the procedure of coalition formation. A learning strategy is introduced to guarantee that the game converges to the Nash equilibrium. We also investigate the condition for the core of the coalitional game being nonempty.

The rest of the paper is organized as follows: Section II describes the mathematical framework within which we deal with the concepts just discussed. The terminology we use in the paper is defined. Section III presents various gain and cost models that can be used in communication networks. In Section IV-B we present the learning strategy that drives

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the dynamic game to the Nash equilibrium and the sufficient condition for nonempty core. A specific gain and cost model is studied in detail. Section V concludes the paper.

II. Problem Formulation

A. Networks

Suppose there are n nodes[1] in the network. Define the set of nodes $N = \{1, 2, \ldots, n\}$. The communication structure of the network is represented by an undirected graph g, where a link between two nodes implies that they are able to directly communicate.

Let g^N represent the *complete graph*, where every node is directly connected to every other node, and let the set $G = \{g | g \subseteq g^N\}$ be the set of all possible graphs. If i and j are directly linked in g, we write $ij \in g$. Let $g + ij$ denote the graph obtained by adding link ij to the existing graph g, where $ij \notin g$ and $g - ij$ denote the graph obtained by severing link ij from the existing graph g where $ij \in g$ (i.e., $g + ij = g \cup \{ij\}$ and $g - ij = g \setminus \{ij\}$). The set of nodes in graph g is $N(g) = \{i | i \in g\}$ and $n(g)$ is the number of nodes in g.

A communication link is established only if the two end nodes (of the link proposed) agree to collaborate with each other, i.e., they are *directly* connected with each other in g. Once the link is added, two end nodes join one coalition and they agree to forward all the traffics from each other. Note that *indirect* communication between two players require that there is a path connecting them. A path in g connecting nodes i_1 and i_m is a set of distinct nodes $\{i_1, i_2, \ldots, i_m\} \subset N(g)$, such that $\{i_1 i_2, i_2 i_3, \ldots, i_{m-1} i_m\} \subset g$.

The communication structure g gives rise to a partition of the node set into groups of nodes who can communicate with each other. A *coalition* of g is a subgraph $g' \subset g$, where $\forall i \in N(g')$ and $j \in N(g')$, $i \neq j$, there is a path in g' connecting i and j, and for any $ij \in g$ implies $ij \in g'$.

Define $\mathcal{N}_i(g) = \{j \in N | ij \in g\}$ as the neighbor set of node i. Furthermore, a pair of nodes are connected in network g if there is a path between i and j, denoted as $i \overset{g}{\longleftrightarrow} j$. We define $\mathcal{C}_i(g) = \{j \in N | i \overset{g}{\longleftrightarrow} j\}$ as the set of all nodes with whom i communicates either directly or through other nodes.

B. Value functions

Following the work by Jackson and Wolinsky [8], we use a value function to represent the overall value generated by a particular network structure. The value function is the natural extension of the characteristic function from cooperative game theory. In cooperative game theory, the characteristic function depends just on the set of players. On the other hand, the value function (in this paper) depends on the detailed network structure rather than simply on a set of players. In fact, the characteristic function is the special case of the value function (for a cooperative game in partition function form), where the value (of the game) only depends on coalitions rather than network structure. The special case where the value function depends only on the groups of players that are connected, but not on how they are connected, corresponds to cooperation schemes that were first considered by Myerson [9], where the particular structure of the network did not matter, as long as the coalition's members were connected somehow. Myerson started with a transferable utility cooperative game in characteristic function form, and layered on top of that network structures that indicated which players could communicate. A coalition could only generate value if its members were connected via paths in the network.

A value function is a function $v : G \to \mathcal{R}$. For simplicity, we assume the normalization that $v(\emptyset) = 0$. The set of all possible value functions is denoted $\mathcal{V}$. A value function v of a coalition $\mathcal{C}(g)$ is component-wise additive if $\sum_{i \in \mathcal{C}(g)} v_i(g) = v(\mathcal{C}(g))$. It is important to note that different networks that consists of the same set of players may lead to different values. This makes the value function a much richer object than the characteristic function used in cooperative game theory. In most applications, there may be some costs associated to links and thus some difference in total value across networks even if they connect the same sets of players, and so this more general and flexible formulation is more powerful. For instance, a society $N = \{1; 2; 3\}$ may have a different value depending on whether it is connected via the network $g = \{12; 23\}$ the network $g^N = \{12; 23; 13\}$. It is also important to note that the value function can incorporate costs to links as well as benefits. It allows for quite general ways in which costs and benefits may vary across networks as we will discuss in detail in Section III.

C. Network games

A network game is a pair $(N; v)$ where N is the set of players and v is a value function on networks among those players. The use of such games will involve both cooperative and non-cooperative perspectives, as they will be the basis for network formation [10], [6].

We create a dynamic game on the graph g. The players of the game are the users. The actions available to user i are "activate link ij" and "not activate link ij" for each neighboring user j. If the edge ij becomes active, then both users pay the costs and reap the benefits (each his own). Otherwise, neither one receives or pays anything. On the other hand, if an existing edge becomes inactive, the payoff of both players is reevaluated. The *action* of user i is a vector defined as $x_i = (x_{i,1}, \ldots, x_{i,i-1}, x_{i,i}, x_{i,i+1}, \ldots, x_{i,n})$ where $x_{i,j} \in \{0, 1\}$ for each $j \in N$ and $x_{i,i} = 0$. $x_{i,j} = 1$ is interpreted as saying that user i wants to activate the link between i and user j, while $x_{i,j} = 0$ states that i does not directly communicate with user j. The set of all actions of user i is denoted by X_i. Since user i has the opinion of activating or not activating a link with each of the remaining $n - 1$ users, the number of actions of user i is $|X_i| = 2^{n-1}$. The set $X = X_1 \times \cdots \times X_n$ is the action space of all the users. A link ij is formed in network g only if $x_{i,j} = 1$ *and* $x_{j,i} = 1$. Therefore, an action profile

[1] In this paper, the terms node, player and user are interchangeable.

$x = (x_1, \ldots, x_n)$ corresponds to the network g. Figure 1 gives an example of the correspondence between the action profile and the network. There is a link between 1 and 2 because $x_{1,3} = x_{3,1} = 1$, while 2 and 3 are not linked since $x_{3,2} = 0$ even though $x_{2,3} = 1$.

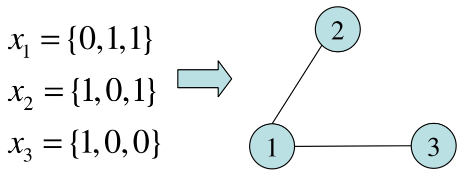

Fig. 1. The correspondence of the action profile and the network

Notice that the game we define here is time-dependent. In other words, the payoff players receive varies over time. This leads us to observe that the dynamics of the game can be separated in rounds of successive coalition expansions (or contractions). That is, in the first round, some edges will be activated based on the initial benefit, which will increase the appropriate benefit for the next round. In the second round, we will examine the new benefit to see if new edges can potentially become active or existing edges may become inactive, and so on.

Therefore, we describe the dynamic coalition formation process as an iterated game. The game is assumed to be repeated in each time period $t = 1, 2, \ldots$. Since our game is played iteratively, we use x_i^t to represent the action i chooses at time t. In the rest of the paper, the payoff of user i at time t is denoted as $v_i(x^t)$. As usual, we let x_{-i}^t denote a tuple of actions taken by all users other than i at time t.

The players' probability of playing action x at time t is denoted as $q^t(x)$, where q^t is a probability distribution on X. We assume that the action each user executes at each time t is independent from the actions of other users, i.e. $q^t(x) = \prod q_i^t(x_i)$, where each q_i^t is a probability distribution on X_i.

When both i and j activate the edge towards each other, they join in a *coalition*. A coalition is a subset of users that is connected in the subgraph induced by the active edges (i.e., the graph $g' = (V, E')$, where E' is the set of active edges). In other words, two users are in the same coalition, if and only if there exists a path of active edges between them. The users in a coalition are the coalition's *members*. The *size* of a coalition is the number of its members. A single isolated user can also be said to be in a trivial coalition of size 1. Define C_i to be the set of users that form the coalition user i belongs to, and let C_i^t denote such a set at time t. If two members of separate coalitions decide to activate the edge between them, then the two coalitions *merge* into one. Suppose user i and user j decide to activate link ij at time t, we have that

$$C_i^t = C_j^t = C_i^{t-1} \cup C_j^{t-1}.$$

D. Allocation rules

Beyond knowing how much total value is generated by a network, it is also critical to keep track about how that value is allocated or distributed among the players. This is captured by the concept of an allocation rule. An allocation rule is a function $Y : G \times V \rightarrow \mathcal{R}^N$ such that $\sum_i Y_i(g, v) = v(g)$ for all v and g. Generally, there will be some natural way in which the value is allocated. It might simply be the utility that the players directly receive, accounting for both the costs and benefits of maintaining their links, which is the model we use in this paper. It might also be the result of some bargaining. Beyond the allocations that come naturally with the network, we are also interested in designing the allocation rule. This might be motivated in a number of ways, including trying to provide the incentives for players to form networks. Thus an allocation rule captures either an allocation that arises naturally or an allocation of value that is imposed. The design of the allocation rule is not discussed in this paper and will be our future work.

It is important to note that an allocation rule depends on both g and v. This includes not only what the network configuration is, but also how the value generated depends on the overall network structure. For instance, consider a network $g = \{12; 23\}$ where the value generated is 1 ($v(g) = 1$). Player 2's allocation might be very different depending on what the values of other networks are. For instance, if $v(\{12; 23; 13\}) = 0 = v(\{13\})$, then 2 is essential to the network and may receive a large allocation. If on the other hand $v(g') = 1$ for all networks, then 2's role is not particularly special.

III. Games in Communication Networks

In this section, we give detailed examples of game models in communication networks. More specifically, we discuss different types of value functions that consist of the benefit nodes gain from joining a coalition and the cost of joining. Autonomic networks depend on collaboration between their nodes for all their functionalities. The nodes, even if modeled as selfish, gain from such collaboration, in the sense that they can accomplish functionality and performance that is impossible to achieve without such collaboration. However, such gains from collaboration do not come for free. There are costs for such collaboration incurred by each node (e.g. energy consumption for forwarding other nodes packets). We consider the tradeoff between such gain and cost in the context of user collaboration in autonomic networks, as a fundamental principle.

A. Gain

As we have discussed, users obtain benefits by joining a coalition. A user's gain is explicitly defined on how he is connected to other users in the coalition. Generally, the gain of user i at time t is $b_i(\mathcal{C}_i^t)$. In what follows, we describe some specific gain functions.

1) Information-theoretic model: It is reasonable to assume that the links formed can sustain a certain information flow, which is proportional to the signal to noise ratio (SNR). In an information theoretic network model, the relationship between the rate of the information flow and the SNR is non-linear. The rate on link ij is given by,

$$R_{ij} = \log\left(1 + \frac{Pl(d_{ij})}{N_0}\right),$$

where P is the transmitter power, l is the loss factor due to diffusion and absorption in the environment, which is a function of the geometric distance between users i and j denoted as d_{ij}, and N_0 is the environmental noise. The formula shows that every pair of neighbors in the network is connected by a link that can sustain a constant information rate. Here we assume that certain scheduling mechanisms, such as TDMA or FDMA, are used to make sure there is no interference caused by simultaneous communication of different nodes in the network. If interference exists, more items need to be added to the denominator in addition to environmental noise.

Given the constant rate of information flow node i receives via link ij, we can define the gain of collaboration as

$$b_{ij} = R_{ij},$$

Thus the gain of node i from joining the coalition is

$$b_i(g) = \sum_{j \in \mathcal{N}_i(g)} R_{ij}.$$

2) Social connection model: This model is studied by Jackson and Wolinsky [8] in the context of social and economic network formation. Assume that nodes always have information sent to other nodes in the network. Thus we assume each node potentially offers benefits V to other nodes per time unit. For instance, V could be the number of bits per time unit each node could provide, which is a function of the link capacity.

The potential benefit is an expected value, which may be reduced during transmissions in the network. The gain of node i is defined as

$$b_i(g) = \sum_{j \in g} V\delta^{r_{ij}-1}, \tag{1}$$

where r_{ij} is the number of hops in the shortest path between i and j (also known as the geodesic distance in graph theory[2]), and $0 \leq \delta \leq 1$ is the communication depreciation rate. If there is no path between i and j, $r_{ij} = \infty$. The gain function gives higher value to paths with smaller number of hops. It captures the fact that more directly connected nodes gain more than nodes far away in terms of geodesic distance. The depreciation can be explained by communication reliability and efficiency due to transmission failures or delay.

B. Cost

On the other hand, activating links is costly.

1) Power cost: For instance, the cost for user i to activate his communication link to user j can be equal to the transmission energy (or power) necessary for i to send data to j. In the networks with fixed transmission power, we have that

$$c_{ij} = P$$

if ij is activated.

In the case of adjustable transmission power, the sender can choose the power so that the signal strength at the receiver is just enough to receive the signal. According to the wireless propagation model, transmission power consumption depends on the geometric distance between i and j, (d_{ij}). Then we can define c_{ij} as a function of d_{ij}

$$c_{ij} = Pd_{ij}^{\alpha},$$

where P is a parameter depending on transmitting power, transmitter/receiver antenna gain and system loss not related to propagation, and α is the path loss exponent depending on the specific propagation environment. Notice that in this cost model, a link can be activated between any pair of nodes by adjusting transmission power. However, a link between two nodes that are faraway introduce very high cost, so the link with high cost will only be activated if the gain from activating it is very high in the coalition formation process.

2) Risk or penalty: The cost can also be something related to risk, or penalty, user i expects if it connects to user j. For instance, if j is highly trusted, the cost of establishing the link is low. Suppose the trust i has on j is t_{ij}, which is a value between 0 and 1, we can define the cost as the inverse of the trust values, that is

$$c_{ij} = \frac{1}{t_{ij}}.$$

C. Value function

The payoff user i has at time t, is defined as

$$v_i(g^t) = b_i(g^t) - \sum_{j \in \mathcal{N}_i^t} c_{ij}.$$

We consider component-wise additive value functions. Thus for coalition C, we have

$$v(C) = \sum_{i \in C} v_i(g).$$

IV. Game Dynamics and Coalition Formation

In this section, we study the dynamics of coalition formation, including convergence of the game and coalitions formed when the game converges.

A. Game dynamics

We pose the following question: *Are there simple strategies that lead our coalition formation to equilibrium?* We introduce here a strategy that is particularly simple and intuitive, where players stochastically adjust their strategies by a reinforcement learning rule. The adjustment is guided by "regret" [11] based on observations from past periods. Users know the past history of all other users they interact with.

We first define the Nash equilibrium of the formation game:

Definition 1: q is a *Nash equilibrium* if no player i can

[2] r_{ij} is the geodesic distance as opposed to the geometric distance d_{ij}

deviate from q and achieve a higher payoff:

$$\forall i, x'_i \in X_i, \quad (2)$$
$$\sum_{x_{-i}\in X_{-i}} v_i(x'_i, x_{-i}) \prod_{j\neq i} q_j(x_j) \leq \sum_{x\in X} v_i(x) \prod_j q_j(x_j).$$

The strategy is as follows: At each period, a player may either choose to continue playing the same action as in the previous period, or switch to other actions with probabilities that are proportional to how much higher his accumulated payoff would have been if he had always made that change in the past. More precisely, the average payoff through time t for user i is

$$\bar{v}_i^t = (1/t) \sum_{1\leq s\leq t} v_i(x_i^t, x_{-i}^t).$$

For each action x'_i, define the average *regret* [11] from not having played x'_i as

$$\bar{r}^t_{i,x'_i} = \sum_{1\leq s\leq t} v_i(x'_i, x^t_{-i}) - \bar{v}^t_i.$$

Then the *regret matching strategy* is defined as the following: in each time period $t+1$, the player i plays either action activate or not activate with a probability proportional to the nonnegative part of his regret up to time t:

$$q_i^{t+1}(x_i) = \frac{(\bar{r}^t_{i,x_i})^+}{\sum_{x'_i\in X_i}(\bar{r}^t_{i,x_i})^+}. \quad (3)$$

Given the probability sequences in time $(q^1, q^2, \dots)$, the game yields a stochastic process that generates sequences of actions $\omega = (x^1, x^2, \dots)$. Such a sequence is a *realization* of the strategy. Let $\phi^t(x)$ be the proportion of times up to t, that each action-tuple x was played. Similarly, for each user i let $\phi^t_{-i}(x_{-i})$ be the proportion of time that users other than i played x_{-i} (that is ϕ_{-i} is the i^{th} marginal of ϕ). The empirical distribution ϕ is a Nash equilibrium if it satisfies Eqn. (2), i.e.

$$\forall i, x'_i \in X_i, \sum_{x_{-i}\in X_{-i}} v_i(x'_i, x_{-i})\phi_{-i}(x_{-i}) \leq \sum_{x\in X} v_i(x)\phi(x). \quad (4)$$

Definition 2: A given realization of game dynamics ω has *no regret* if

$$\limsup_{t\to\infty} \bar{\mathbf{r}}^t_i \leq 0.$$

B. Convergence of the formation game

A similar regret matching strategy was proposed and studied by Hart and Mas-Colell [12] for a finite N-person non-cooperative game. The differences between the games we study in this paper and those in [12] are that our formation game is played on networks (i.e. users only play games with their neighbors in the network) and more importantly the payoff of each user is changing over time since it depends on the coalition structure that is changing as users activate or deactivate links. Nevertheless, inspired by the results of Hart and Mas-Colell, we have the following result regarding the convergence of dynamic games on the graph.

Theorem 1: Given that all players use the regret matching strategy, the empirical distribution ϕ converges almost surely to the set of Nash equilibria.

The basic idea behind regret matching is to push the nonnegative part of regret to zero. Therefore, in the long run, the regret matching strategy almost surely yields no regret and the empirical distribution converges to a superset of the equilibria. Note that convergence to the *set* of Nash equilibria does not imply that the sequence ω converges to *a point*.

We will need the following lemma to prove Theorem 1.

Lemma 2: Suppose that at every time $t+1$ player i chooses actions according to the probability vector $q_i^{t+1}(x_i)$ defined in Eqn. 3. Then player i yields no regret almost surely against every possible sequence of actions by the other players as $t\to\infty$.

Proof: We consider a specific link of player i, say ij. We have that $x_{ij} \in \{0,1\}$. At each time t, denote player i's average payoff as $\bar{v}^t$ (we omit index i in the notations of this proof). Let $\bar{v}^t_0$ be the average payoff that would have resulted if i had always chosen to not activate link ij in every time period. The regret from not having played action 0 is therefore $\bar{r}^t_0 = \bar{v}^t_0 - \bar{v}^t$. Similarly, the regret for not having played 1 is $\bar{r}^t_1 = \bar{v}^t_1 - \bar{v}^t$. The regret vector at time t is $\bar{r}^t = (\bar{r}^t_0, \bar{r}^t_1)$.

At time $t+1$, no matter what j plays and what the coalition structure is, the regret from playing 0 is the negative of the regret from playing 1, since one is $r_0^{t+1} = v_1^{t+1} - v_0^{t+1}$ and the other is $r_1^{t+1} = v_0^{t+1} - v_1^{t+1}$. Define $r_0^{t+1} = s^{t+1}$, then the regret vector is $\mathbf{r}^{t+1} = (s^{t+1}, -s^{t+1})$. The value of s^{t+1} depends on j's choice at $t+1$ and also the coalition structure after time t, which is unknown. We have that the probability for i to choose 0 or 1 at $t+1$ is q_0^{t+1} and q_1^{t+1} respectively. Then the expected regret at time $t+1$ is

$$\mathrm{E}[\mathbf{r}^{t+1}] = s^{t+1}(q_1^{t+1}, -q_0^{t+1}). \quad (5)$$

Then we have the expected average regret at time $t+1$ as

$$\mathrm{E}[\bar{\mathbf{r}}^{t+1}] = \frac{t\bar{\mathbf{r}}^t + \mathrm{E}[\mathbf{r}^{t+1}]}{t+1}. \quad (6)$$

According to the definition of q:

$$\frac{q_0^{t+1}}{q_1^{t+1}} = \frac{(\bar{r}^t_0)^+}{(\bar{r}^t_1)^+}. \quad (7)$$

We have that

$$\mathbf{E}[\mathbf{r}^{t+1}]\cdot(\bar{\mathbf{r}}^t)^+ = s^{t+1}(q_1^{t+1}(\bar{r}^t_0)^+ - q_0^{t+1}(\bar{r}^t_1)^+) = 0. \quad (8)$$

Consider the case where the average regrets are positive at time t, i.e., $\bar{\mathbf{r}}^t = (\bar{\mathbf{r}}^t)^+$. We have that

$$\begin{aligned}
&|\mathrm{E}[\bar{\mathbf{r}}^{t+1}]|^2 \\
= \;& \left(\frac{t}{t+1}\bar{\mathbf{r}}^t + \frac{1}{t+1}\mathrm{E}[\mathbf{r}^{t+1}], \frac{t}{t+1}\bar{\mathbf{r}}^t + \frac{1}{t+1}\mathrm{E}[\mathbf{r}^{t+1}]\right) \\
= \;& \frac{t^2}{(t+1)^2}|\bar{\mathbf{r}}^t|^2 + \frac{1}{(t+1)^2}|\mathrm{E}[\mathbf{r}^{t+1}]|^2 \\
\leq \;& |\bar{\mathbf{r}}^t|^2 \text{ given that } t \text{ is sufficiently large.}
\end{aligned}$$

The second equation holds because of Eqn. (8), Similarly, we can also prove that $|\left(\mathrm{E}[\bar{\mathbf{r}}^{t+1}]\right)^{+}| \leq |(\bar{\mathbf{r}}^{t})^{+}|$ when the average regrets are not all positive at time t.

Thus, we have that the nonnegative part of the expected average regret at time $t+1$ ($\left(\mathrm{E}[\bar{r}^{t+1}]\right)^{+}$) is less than the nonnegative part of the average regret at t ($(\bar{r}^{t})^{+}$). It follows that the vector $\left(\mathrm{E}[\bar{r}^{t+1}]\right)^{+}$ converges almost surely to zero. ■

Given the above Lemma, we can prove Theorem 1 as follows.

Proof: The different coalitions formed by activating or deactivating links can be modelled as a Markov chain with finite states. The regret values change while the game evolves from one state to another state of the Markov chain. Since the distribution q depends on regret values, changes in regret values introduce mutations in the model. Given nonzero mutations for each state of the Markov chain, we have that the Markov chain is irreducible and aperiodic. Therefore, it has a unique corresponding stationary distribution. The results of Harsanyi and Selten [13] and Kandori, et al. [14] show that by letting the mutation probability go to 0 (in our case the player yields no regret), the game converges to equilibrium. ■

C. Stability of the coalition

The main concern in cooperative games is how the total payoff from a partial or complete cooperation of the players is divided among the players. A *payoff allocation* is a vector $\mathbf{Y} = (Y_i)_{i\in N}$ in $\mathbf{R}^N$, where each component Y_i is interpreted as the payoff allocated to player i. We say that an allocation $\mathbf{Y}$ is *feasible for a coalition* S iff $\sum_{i\in S} Y_i \leq v(S)$.

When we think of a reasonable and stable payoff, the first thing that comes to mind is a payoff that would give each coalition at least as much as the coalition could enforce itself without the support of the rest of the players. In this case, the players could not get better payoffs if they form separate coalitions instead of forming the *grand coalition* N. The set of all these payoff allocation of the game (N, v) is called the *core* and is formally defined as the set of all n-vectors $\mathbf{Y}$ satisfying the linear inequality:

$$Y(S) \geq v(S) \quad \forall S \subset N, \tag{9}$$

$$Y(N) = v(N), \tag{10}$$

where $Y(S) = \sum_{i\in S} Y_i$ for all $S \subset N$. Apparently, the core is a much stronger stable concept than the Nash equilibrium. However, it is known that the core may possibly be empty. We present, in what follows, a sufficient condition for the core of the formation game to be nonempty; thus all players belong to the grand coalition (i.e., all players in the network are connected to each other).

We first give the definition of a family of very common games: *convex games*.The convexity of a game can be defined in terms of the marginal contribution of each player. $d_i : 2^N \to \mathbf{R}$ of v with respect to player i is

$$d_i(S) = \begin{cases} v(S \cup \{i\} - v(S) & \text{if } i \notin S \\ v(S) - v(S \setminus \{i\}) & \text{if } i \in S \end{cases}$$

A game is said to be *convex*, if for each $i \in N$, $d_i(S) \leq d_i(T)$ holds for any coalitions $S \subset T$. It has been proven that the core of a convex game is nonempty [15]. Thus as long as the coalition formation game is convex, the core is nonempty. We have the following result.

Theorem 3: If $\forall i, j,\ v_{ij} + v_{ji} \geq 0$, the core of the formation game is nonempty.

Proof: In the formation game, $d_i(S) = \sum_{j\in S, j\neq i}(v_{ji} + v_{ij})$. Take two sets S and T where $S \subset T$,

$$d_i(T) - d_i(S) = \sum_{j\in T\cap S^c} (v_{ji} + v_{ij}) \geq 0$$

Therefore, the formation game is a convex game. Its core is nonempty. ■

D. Case study

In this section, we study the network formation of a specific model where the gain $b_i(g) = |C^t(i) - 1|$, i.e., users benefit from connecting to as many other users as possible, directly (one-hop) or indirectly (multi-hop, through other users), and the cost c_{ij} is modelled as a random variable with an exponential probability distribution with parameter λ. The exponential distribution is a natural choice when we only know that the average cost is $1/\lambda$, since the exponential distribution has the maximum entropy. For instance, the expected risk of establishing a link is known ahead of time. First we derive some general properties of this model.

Corollary 4: All coalitions formed are trees.
Since the benefit b_i is a function of the size of the coalition user i belongs to, and not of the structure, any extra links between nodes that are already connected only add cost, and do not add to the benefit.

Based on Theorem 3, we can derive the lower bound on the probability of which the core of this simple coalitional game is nonempty.

Corollary 5: The probability that the game has nonempty core is greater than or equal to $(1 - e^{-4\lambda})^{N(N-1)}$.

Proof: Consider user i and j. The cost of activating link ij is an exponentially distributed random variable with parameter λ, denoted as c_{ij} for i and c_{ji} for j. The mimimum benefit i and j gets by activating ij is equal 2, i.e., i and j form a two-user coalition. The sufficient condition for nonempty core is that

$$v_{ij} + v_{ji} \geq 0. \tag{11}$$

Thus we have that

$$2 - c_{ij} + 2 - c_{ji} \geq 0. \tag{12}$$

Since c_{ij} and c_{ji} are independent exponential random variables, we have that the probability the inequality (12) is satisfied is $1-e^{-4\lambda}$. Since we have in total $N(N-1)$ pairs of users in the network, the core is nonempty if all these pairs satisfy inequality (11). Therefore, the lower bound on the probability of the core being nonempty is $(1-e^{-4\lambda})^{N(N-1)}$. ■

We further investigate coalitions formed given a specific topology and a specific choice of cost weights. We are inter-

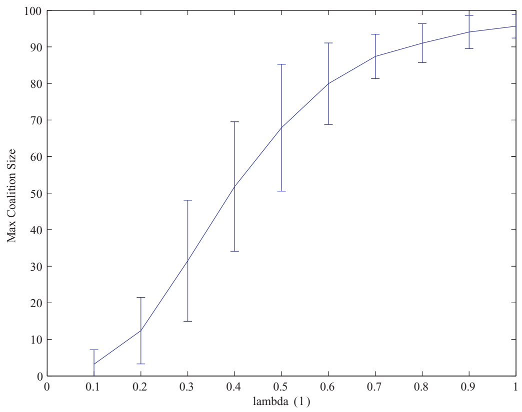

Fig. 2. Maximum coalition size as a function of the parameter λ. Edge weights are λ-exponentially distributed. Averages of 100 simulations are plotted, along with one standard deviation on each side of the average.

ested in the size of the largest coalition. We run simulations to study the games at equilibrium. In our simulations, we consider 100 users in a 10×10 square grid topology. Each player is connected to his N, S, W, and E neighbors, except, of course, the players on the boundary of the square. In Figure 2, we show that as the parameter λ increases, the maximum coalition size increases.

V. Conclusions

In this paper, we study autonomic networks which rely on the collaboration of participating nodes. We model the games played by users as a dynamic game. The dynamics of coalition formation proceed via nodes that interact strategically and adapt their behavior to the observed behavior of others. We present conditions that the coalition formed is stable in terms of Nash stability and the core.

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