

AERIAL PLATFORM PLACEMENT ALGORITHMS TO SATISFY CONNECTIVITY, CAPACITY AND SURVIVABILITY CONSTRAINTS IN WIRELESS AD-HOC NETWORKS

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ABSTRACT

In this paper, we address the problem of establishing full connectivity and satisfying required traffic capacity between disconnected clusters in large wireless ad-hoc ground networks by placing a minimum number of advantaged high flying Aerial Platforms (APs) as relay nodes at appropriate places. We also extend the connectivity solution in order to make the network survivable to a single AP failure. The problem of providing both connectivity and required capacity between disconnected ground clusters is formulated as a summation-form clustering problem with inter-AP distance constraints that make the AP network fully connected and with complexity costs that take care of cluster to AP capacity constraints. The resultant clustering problem is solved using Deterministic Annealing to find (near) globally optimal solutions for the minimum number and locations of the APs to establish full connectivity and provide required traffic capacity between disconnected clusters. In order to make the network single AP survivable, the connectivity solution is extended so that each AP connects to at least two neighboring APs and each ground cluster connects to at least two APs. We establish the validity of our algorithms by comparing it with optimal exhaustive search algorithms and show that our algorithms are near-optimal for the problem of establishing connectivity between disconnected clusters.

I. INTRODUCTION

Wireless Mobile Ad Hoc Networks (MANETs), where nodes form and maintain a wireless multihop network without any central infrastructure, are becoming popular both in the commercial and the military world. But it is highly probable that a MANET contains nodes that are disconnected from each other. One of the methods suggested to improve connectivity, capacity, robustness, and survivability of MANETs is to use Aerial Platforms (APs) as relays in the network.

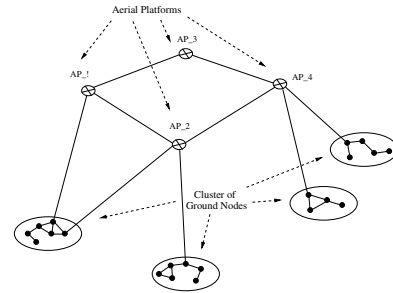


Fig. 1. Network with four partitions and four connecting APs.

In this paper, we first look at the problem of providing (basic) connectivity between disconnected ground clusters and satisfying required traffic capacity between these clusters by placing a minimum number of APs at appropriate places to act as relay nodes. Since Aerial Platforms are scarce and expensive resources, the goal is to find the minimum number of APs and their locations so that the resultant network (both between ground nodes and APs) is connected and there are enough pathways to support the required inter-cluster capacity (see Figure 1). In [1], the authors use a deterministic annealing (DA) clustering approach ([2]) to find near-optimal solutions for the minimum number of APs and their location so that at least one node from each ground cluster is connected to at least one AP. We extend the approach of [1] in two ways: *a)* include AP-AP communication distance constraints so that not only are the clusters connected to the APs but also the APs form a connected network; *b)* the APs connected to each cluster are capable of supporting the required capacity from each cluster to other clusters with maximum AP-cluster link utilization. We then extend the basic connectivity solution in order to make the resultant network single AP survivable.

The paper is organized as follows. Section II describes our scenario and the assumption made. Section III explains our formulation of the basic connectivity problem and the inter-cluster capacity problem in the framework of a constrained clustering problem with complexity costs. Sec-

tion IV explains the DA solution to the basic connectivity and inter-cluster problems and gives a brief review of the algorithm used. Section V extends the DA solution to make the network single AP survivable. Section VI presents the results of the DA algorithm for basic connectivity, inter-cluster capacity, and single AP survivable network. We also compare our connectivity results with an exhaustive grid search algorithm. Finally, we conclude in section VII.

II. SCENARIO AND ASSUMPTIONS

Let the ground nodes and the APs have identical omnidirectional radios with free space communication (where the signal decays as $1/R^2$, with R being the distance) possible if the distance between two radios is less than R_2 . Since the ground nodes communicate with one another in an environment (indirect reflections, etc.) where the signal decays as $1/R^\alpha$, where α is greater than 2 (suburban decay is as $1/R^4$), we assume that the ground nodes can communicate with each other if their distance is less than R_0 (with $R_0 < R_2$). Assume that the ground network has N nodes (with positions $G = \{g_i, i = 1, \dots, N\}$) forming M clusters where the nodes within each cluster can communicate with each other and the nodes in different clusters cannot communicate with one another. Each cluster is represented by $K_j, j = 1$ to M . Also assume that all of the ground nodes, $g_i (i = 1, \dots, N)$, have the same altitude (of 0). This assumption basically keeps the problem in \mathcal{R}^2 and is a reasonable approximation for most practical cases.

Let each AP fly at a maximum cruising altitude of h in a holding pattern above the scenario. Since the AP-AP and AP-ground node communication can be modeled as that of free space, it is assumed that the AP-AP or AP-ground node communication can take place if the distance between the nodes is less than R_2 . Since all APs fly at a constant altitude h , the connectivity problem can be reduced to \mathcal{R}^2 , with the positions of the APs projected onto the ground and denoted by a_k (with $A = \{a_k, k = 1, \dots, L\}$ assuming L APs). This results in a maximum AP-ground node communication distance of $R_1 = \sqrt{R_2^2 - h^2}$ with the AP-AP maximum communication distance being R_2 .

Assume that the maximum AP-AP and AP-ground node link capacity is C_{max} . Let the total capacity required from source cluster K_i to destination cluster K_j be C_{ij} with C_{ii} taken to be 0. Hence the total capacity (C_i) of the links going out and coming into cluster K_i is $C_i = \sum_{j=1}^K (C_{ij} + C_{ji})$. We need to have $C_i \leq C_{max} (\forall i = 1, \dots, M)$ as the maximum AP-ground node capacity is C_{max} .

III. FORMULATION OF BASIC CONNECTIVITY AND INTER-CLUSTER CAPACITY PROBLEM

If the baseline ground scenario is disconnected, Aerial Platforms can be used to establish connectivity and pro-

vide required capacity. We formulate the basic connectivity problem as a constrained clustering problem ([3], [4]) with a summation form distortion function ($D(K, A)$) involving the distances between the ground clusters (K) and the APs (A) and a summation form cost function ($C_1(A)$) involving only the distances between the APs (A). The capacity constraints, including maximizing the AP-cluster link utilization, are handled by adding a complexity cost function $C_2(p(A))$ ([5]) that only depends on the *assignment probabilities* $p(a_i)$ of the APs; and relating the prior probabilities $p(K_i)$ of each cluster K_i to be proportional to C_i . The resultant clustering problem is then solved using Deterministic Annealing (DA) to obtain near-optimal solutions.

A. Deterministic Annealing

Deterministic Annealing ([2]) is a method for clustering where a large number of data points, denoted by x 's, (in our problem, the various ground clusters) need to be assigned to a small number of centers, denoted by y 's, (in our problem, the various APs) such that the average distortion function is minimized. The average distortion can be written as $D = \sum_x p(x)d(x, y(x))$, where $p(x)$ is the prior probability of data point x . The DA approach tries to avoid local minima by turning the hard clustering problem (where a data point is associated with only one center) into a soft/fuzzy clustering problem (where each data point can be associated to many centers via its *association probabilities* $p(y|x)$) and then minimizing the distortion at various levels of randomness measured by the Shannon entropy $H(X, Y)$. Hence the original distortion function is re-written as $D = \sum_x p(x) \sum_y p(y|x)d(x, y)$ where the *assignment probability* $p(y) = \sum_x p(x)p(y|x)$, measures the percentage of data points assigned to a center y . The objective function that DA minimizes is $F = D - TH$ or $F = D - TH(Y|X)$ at various values of temperature T starting from high temperature and then slowly decreasing the temperature.

B. Basic Connectivity Problem

In order to connect the various ground clusters to the APs while ensuring that the APs form a connected network, we need to find the minimum number of APs L and their positions on the ground, a_k , (with $A = \{a_k, k = 1, \dots, L\}$) such that: *a*) At least one node from each cluster is within a radius of R_1 from an AP (see Figure 2); and *b*) The AP locations a_k are within R_2 from each other (i.e., the APs form a connected graph; see Figure 2). Assuming that the APs are numbered from 1 to L , we can make sure that the APs form a connected network by ensuring that any AP numbered j is connected to atleast one lower numbered

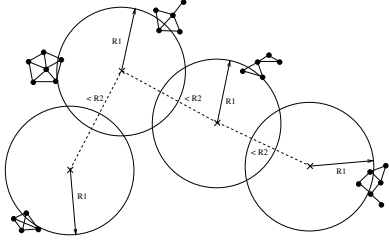


Fig. 2. Aerial Platform Placement.

AP i , where $i < j$. This is used in the DA solution where when we add a new AP, we make sure that it is connected to at least one of the previously added APs. Hence the connectivity problem can be stated as:

$$\begin{aligned} & \text{Minimize } L \\ & \text{subject to} \\ & \exists a_1, \dots, a_L; \quad \max_{j \in \{1, \dots, M\}} \min_{\substack{g \in K_j \\ i \in \{1, \dots, L\}}} \|g - a_i\| \leq R_1 \\ & \text{and,} \quad \max_{l \in \{2, \dots, L\}} \min_{m < l} \|a_l - a_m\| \leq R_2 \end{aligned}$$

where $\|g - a\|$ is the l^2 -norm between points g and a on the ground. Finding the exact solution to the problem above involves an exhaustive search on the different ways in which nodes can be selected from each cluster and the ways clusters can be grouped together for coverage by a single AP all the while making sure that the APs are connected to each other. This problem is NP-hard as it is a generalization of the Euclidean *disk-cover* problem [6]. Hence using the approximation,

$$\max(s_1, \dots, s_n) \cong (s_1^\alpha + \dots + s_n^\alpha)^{\frac{1}{\alpha}} \quad \text{for large } \alpha \quad (1)$$

we can convert the AP-ground node and AP-AP constraints into a summation form,

$$\begin{aligned} & \text{Minimize } L \\ & \text{subject to} \\ & \exists a_1, \dots, a_L; \quad \sum_{j=1}^M d_1(K_j, a_{u_1(j)}) \leq R_1^\alpha \\ & \text{and,} \quad \sum_{l=2}^L d_2(a_l, a_{u_2(l)}) \leq R_2^\beta \end{aligned}$$

for large α and β , where,

$$d_1(K_j, a_i) = \min_{g \in K_j} \|g - a_i\|^\alpha$$

$$d_2(a_l, a_m) = \min_{m < l} \|a_l - a_m\|^\beta$$

$$u_1(j) : \{1, \dots, M\} \rightarrow \{1, \dots, L\}$$

is the function that assigns an AP to every cluster.

$$u_2(l) : \{2, \dots, L\} \rightarrow \{1, \dots, L-1\}$$

is the function that assigns the closest lower numbered AP to an AP.

Within the framework of constrained clustering ([3], [4]), the distortion function between the ground nodes and the APs is given by $D(K, A) = \sum_{j=1}^M d_1(K_j, a_{u_1(j)})$ and the cost function among the APs is given by $C_1(A) = \sum_{l=2}^L d_2(a_l, a_{u_2(l)})$.

C. Capacity Constraints

In order to ensure that the capacity required by a cluster K_i to communicate with other clusters (i.e., C_i) is satisfied by the APs within communication range of the cluster, we need to ensure that the capacity supported by an AP, ($C_{ap}(j) \triangleq \sum_{i=1}^M C_i I(\text{AP } j \text{ is associated with cluster } i)$), is less than the maximum link capacity C_{max} . Since in the clustering formulation, we can have a single cluster K_i associated with different APs via its association probabilities $p(a_j|K_i)$, we rewrite $C_{ap}(j)$ as $C_{ap}(j) = \sum_{i=1}^M C_i p(a_j|K_i) \leq C_{max}$. Denoting the AP-cluster link utilization as $u(j) = C_{ap}(j)/C_{max}$, in order to maximize the sum of the AP-cluster link utilizations, we would like to maximize $\sum_{j=1}^L u(j)$.

In order to satisfy the capacity constraints from each cluster K_i to all other clusters, we let the cluster prior probability be set to $p(K_i) = C_i / \sum_{j=1}^M C_j$ and add a complexity cost function $C_2(p(a_k)) = 1/p(a_k)^s$ that only depends on the assignment probabilities of the APs. For high values of s , the cost value for small $p(a_k)$ (i.e., $1/p(a_k)^s$) blows up and the end resultant solution ([5]) tends to be load balanced, i.e., $p(a_k) = 1/L, \forall k = 1, \dots, L$. But

$$\begin{aligned} p(a_k) &= \sum_{i=1}^M p(K_i) p(a_k|K_i) \\ &= \sum_{i=1}^M \left(\frac{C_i}{\sum_{j=1}^M C_j} \right) p(a_k|K_i) \\ \Rightarrow p(a_k) \sum_{j=1}^M C_j &= \sum_{i=1}^M C_i p(a_k|K_i) = C_{ap}(k) \end{aligned}$$

Hence we stop adding APs when the maximum of $p(a_k) \sum_{j=1}^M C_j$ over all the APs becomes less than the maximum link capacity C_{max} . Since all the $p(a_k)$'s are approximately equal, we also tend to maximize the sum of the AP-cluster link utilizations.

IV. DETERMINISTIC ANNEALING SOLUTION

The overall distortion function D including the AP-AP connectivity constraints and the cluster capacity constraints is given by:

$$\begin{aligned} D &= \sum_{i=1}^M p(K_i) \sum_{j=1}^L p(a_j|K_i) [d_1(K_i, a_j) + \eta C_2(p(a_j))] \\ &\quad + \lambda \sum_{l=2}^L d_2(a_l, a_{u_2(l)}) \end{aligned}$$

The deterministic annealing algorithm tries to minimize the objective function $F = D - T H(A|K)$ where

$$H(A|K) = - \sum_{i=1}^M p(K_i) \sum_{j=1}^L p(a_j|K_i) \log p(a_j|K_i).$$

Minimizing F with respect to the association probabilities $p(a_j|K_i)$ with the additional constraints that $p(a_j) = \sum_{i=1}^M p(K_i)p(a_j|K_i)$ and $\sum_{j=1}^L p(a_j|K_i) = 1$ gives the Gibbs distribution:

$$p(a_j|K_i) = \frac{\exp\left(-\frac{d_1(K_i, a_j) + \eta C_2(p(a_j)) + \eta p(a_j) \frac{dC_2(p(a_j))}{dp(a_j)}}{T}\right)}{Z_{K_i}}$$

where

$$Z_{K_i} = \sum_{j=1}^L \exp\left(-\frac{1}{T}(d_1(K_i, a_j) + \eta C_2(p(a_j)) + \eta p(a_j) \frac{dC_2(p(a_j))}{dp(a_j)})\right)$$

The corresponding minimum F^* of F is obtained by plugging the values for $p(a_j|K_i)$ into F to obtain:

$$F^* = -T \sum_{i=1}^M p(K_i) \log Z_{K_i} - \eta \sum_{j=1}^L p^2(a_j) \frac{dC_2(p(a_j))}{dp(a_j)} + \lambda \sum_{l=2}^L d_2(a_l, a_{u_2(l)})$$

The optimal AP locations a_k are given by minimizing F^* leading to the following expression involving the gradient of a_k that needs to be set to zero:

$$\sum_{j=1}^M p(K_j, a_k) \nabla_{a_k} (d_1(K_j, a_k)) + \lambda \nabla_{a_k} \left(\sum_{l=2}^L d_2(a_l, a_{u_2(l)}) \right)$$

This leads to two equations, one for the x coordinate of a_k (i.e., x_{a_k}) and another for the y coordinate of a_k (i.e., y_{a_k}):

$$x_{a_k} = \frac{\alpha \sum_{j=1}^M d1X_{numr}(K_j, a_k) + \lambda \beta (d2X_{numr}(a_k))}{\alpha \sum_{j=1}^M d1denr(K_j, a_k) + \lambda \beta (d2denr(a_k))}$$

$$y_{a_k} = \frac{\alpha \sum_{j=1}^M d1Y_{numr}(K_j, a_k) + \lambda \beta (d2Y_{numr}(a_k))}{\alpha \sum_{j=1}^M d1denr(K_j, a_k) + \lambda \beta (d2denr(a_k))}$$

where

$$d1X_{numr}(K_j, a_k) = x_{K_j} p(K_j) p(a_k|K_j) d_1(K_j, a_k)^{1-2/\alpha}$$

$$d1Y_{numr}(K_j, a_k) = y_{K_j} p(K_j) p(a_k|K_j) d_1(K_j, a_k)^{1-2/\alpha}$$

$$d1denr(K_j, a_k) = p(K_j) p(a_k|K_j) d_1(K_j, a_k)^{1-2/\alpha}$$

$$d2X_{numr}(a_k) = x_{a_{u_2(k)}} d_2(a_k, a_{u_2(k)})^{1-2/\beta} + \sum_{l>k} I(u_2(l) = k) x_{a_l} d_2(a_l, a_k)^{1-2/\beta}$$

$$d2Y_{numr}(a_k) = y_{a_{u_2(k)}} d_2(a_k, a_{u_2(k)})^{1-2/\beta} + \sum_{l>k} I(u_2(l) = k) y_{a_l} d_2(a_l, a_k)^{1-2/\beta}$$

$$d2denr(a_k) = d_2(a_k, a_{u_2(k)})^{1-2/\beta} + \sum_{l>k} I(u_2(l) = k) d_2(a_l, a_k)^{1-2/\beta}$$

For $k = 1$, $d_2(a_k, a_{u_2(k)}) = 0$, so that the first term in $d2X_{numr}(a_k)$, $d2Y_{numr}(a_k)$, and $d2denr(a_k)$ is not present.

A. Algorithm

We start with an initial temperature $T = T_{init}$ and $\lambda = 0$ to get the unconstrained clustering solution for that T (i.e., taking into account only the connectivity between the APs and ground clusters). At a given T , we then gradually increase λ and optimize until the maximum of the minimum inter-node distance between an AP and its lower numbered APs is just less than R_2 . We then reduce the temperature T and repeat the procedure of increasing λ from 0. The temperature T is progressively reduced until all the clusters are covered by at least one AP and the capacity constraints are satisfied, i.e., $\max_{k=1}^L p(a_k) \sum_{j=1}^M C_j \leq C_{max}$.

At each iteration (i.e., fixed T and fixed λ), the association probabilities $p(a_i|K_j)$ are first calculated, then the assignment probabilities $p(a_i)$ are calculated, and finally the optimal AP locations a_i are determined until there is convergence. If after a fixed number of temperature reduction iterations, either all the clusters are not covered or the cluster capacity constraints are not satisfied, then the number of APs is increased. This is done by choosing the AP center i with either farthest associated groups or maximum $p(a_i)$ and adding a small perturbation to its current location, and then dividing its probability $p(a_i)$ equally between the new and old center.

$$p(a_i) = p(a_i)/2; \quad p(a_{L+1}) = p(a_i); \quad L = L + 1$$

If a new center is really needed, then the two centers move apart from each other, else they merge again after a few steps. This is checked by finding the distance between the new and old centers after a couple of temperature reduction iterations and merging them if the distance is less than a threshold.

V. EXTENSION FOR AP SURVIVABILITY

APs can fail (e.g., crashing, being shot down) and hence we would like to have a network of APs such that even if one of the APs fail, the resultant AP-ground cluster network is still connected. A network is called single node survivable

if given that a node and all links to it are removed, a path still exists between any two surviving nodes. Our goal then is to design a network of connected APs that connect all the ground clusters while also being single AP survivable.

A connected network of APs and ground clusters becomes single AP survivable if the following two conditions hold: 1) Each ground cluster is connected to atleast two APs; and 2) Each AP is connected to atleast two neighboring APs. For a particular ground cluster to maintain connectivity with the AP-AP network despite a single AP failure, it needs to be connected to atleast two APs. Now consider a ground cluster connected to only 2 APs. If one of the APs fail and the other AP is only connected to the AP-AP network through the failed AP, then that AP becomes disconnected from the AP network. Hence if every AP is connected to atleast two APs, then the AP-AP connectivity is guaranteed even when one of the AP fails. Thus the two conditions together are sufficient to make the network single AP survivable.

A. Ground Cluster Connected to Two APs

In order to connect a ground cluster to two APs, we extend the DA algorithm as in [1] by modifying the optimal association probabilities $p(a_i|K_j)$ calculated during each iteration of the DA algorithm (see section IV). The association probability $p(a_i|K_j)$ indicates the influence of cluster K_j in determining the AP position a_i . In the DA algorithm, the association probabilities of a cluster start from a uniform distribution at high temperatures (where a cluster equally influences every AP) and converge at low temperatures to a vector with a one and all zeros (hard clustering), where, each cluster affects only one AP. Hence in order for a cluster to be connected to L_i (here $L_i = 2$) APs, at low temperatures, the association probabilities of this cluster to the L_i APs should be large enough to influence the location of the L_i APs. In order to achieve this goal, at each iteration of the DA algorithm, the calculated association probabilities $\mathbf{p}(A|K_i)$ of a cluster K_i to the different AP centers ($A = [a_1, \dots, a_L]$) are adjusted so that the highest L_i probabilities are made equal. This is done by ordering the values of the association probability vector $\mathbf{p}(A|K_i)$ from largest to smallest probability values ($\mathbf{p}(A|K_i) = [p_1, p_2, \dots, p_L]$) and then adjusting the largest $L_i = 2$ probabilities to be the average of the first L_i different probabilities, i.e.,

$$\mathbf{p}(A|K_i) = \left[\frac{1}{L_i} \sum_{j=1}^{L_i} p_j, \dots, \frac{1}{L_i} \sum_{j=1}^{L_i} p_j, p_{L_i+1}, \dots, p_L \right]$$

B. AP Connected to Two Neighboring APs

In order to ensure that an AP l is connected to two neighboring APs and not just one, we extend the con-

nectivity formulation of section III-B (where each AP is constrained to connect to nearest previously added AP) by adding additional AP-AP distance constraints for each AP that is not already constrained to connect to two APs. The additional AP-AP distance constraint for an AP l is of the form: $\min_{m \neq u_2(l)} \|a_l - a_m\| \leq R_2$.

We want the maximum of the above additional AP-AP constraints and the maximum of the AP-AP constraints for basic (single) AP-AP connectivity (as in III-B) to be less than or equal to R_2 . Using the approximation for the maximum as in equation 1, we convert the connectivity constraints into a summation form as given below:

$$\begin{aligned} & \text{Minimize } L \\ & \text{subject to, } \exists a_1, \dots, a_L; \\ & \sum_{j=1}^M d_1(K_j, a_{u_1(j)}) \leq R_1^\alpha \\ \text{and, } & \sum_{l=2}^L d_2(a_l, a_{u_2(l)}) + \\ & \sum_{l=L}^1 I \left(\begin{array}{l} l \text{ needs addl.} \\ \text{conn. constraint} \end{array} \right) d_3(a_l, a_{u_3(l)}) \leq R_2^\beta \end{aligned}$$

for large α and β , where the functions d_1 , d_2 , u_1 , and u_2 are as defined in section III-B and,

$$\begin{aligned} d_3(a_l, a_m) &= \min_{m \neq u_2(l)} \|a_l - a_m\|^\beta \\ u_3(l) &: \{1, \dots, L\} \rightarrow \{1, \dots, L\} - \{u_2(l)\} \\ &\text{is the function that assigns the closest} \\ &\text{AP not equal to } u_2(l) \text{ to an AP.} \end{aligned}$$

Starting from the last AP added (i.e., AP L), if that AP does not already have AP-AP distance constraints to two other APs (either via constraints for basic connectivity through $d_2(a_l, a_{u_2(l)})$ or via previously added $d_3(a_m, a_{u_3(m)=l})$), we add an additional AP-AP distance constraint $d_3(a_l, a_{u_3(l)})$.

C. Changes to DA Solution

The overall distortion function D for the AP to be connected to two neighboring APs (with the cluster capacity constraints) is modified from that in section IV and is given by:

$$\begin{aligned} D &= \sum_{i=1}^M p(K_i) \sum_{j=1}^L p(a_j|K_i) [d_1(K_i, a_j) + \eta C_2(p(a_j))] \\ &+ \lambda \left[\sum_{l=2}^L d_2(a_l, a_{u_2(l)}) + \right. \\ &\left. \sum_{l=L}^1 I \left(\begin{array}{l} l \text{ needs addl.} \\ \text{conn. constraint} \end{array} \right) d_3(a_l, a_{u_3(l)}) \right] \end{aligned}$$

The equation for the optimal association probabilities remains the same as in section IV but the equation for the

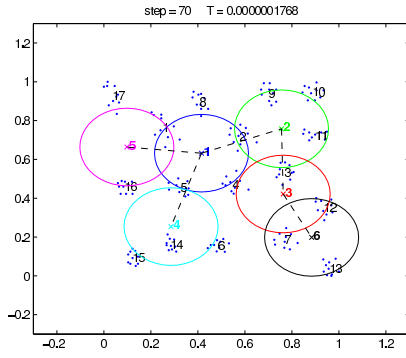


Fig. 3. AP Placement with AP-ground node connectivity and AP-AP connectivity.

optimal AP locations a_k changes to include the additional AP-AP distance constraints $d_3(a_l, a_{u_3(l)})$ leading to the following expression for the gradients of a_k that need to be set to zero.

$$\sum_{j=1}^M p(K_j, a_k) \nabla_{a_k} (d_1(K_j, a_k)) + \lambda \nabla_{a_k} \left(\sum_{l=2}^L d_2(a_l, a_{u_2(l)}) \right) + \lambda \nabla_{a_k} \left(\sum_{l=L}^1 I \left(\begin{array}{l} l \text{ needs addl.} \\ \text{conn. constraint} \end{array} \right) d_3(a_l, a_{u_3(l)}) \right)$$

The DA algorithm proceeds as in section IV-A except for two changes. At a given T , the value of λ is increased till the distance from each AP to atleast two other APs is less than R_2 . Also at each iteration (i.e., for fixed T and fixed λ), the association probabilities $p(a_i|K_j)$ are first calculated according to the formula given in section IV and are then changed as per section V-A so that a ground cluster is connected to two APs.

VI. RESULTS

A. Basic Connectivity Constraints

To test the basic connectivity solution, we fix the inter-ground node communication distance R_0 to 0.1 and AP-ground node communication distance R_1 to 0.2. We set AP-AP communication distance to $R_2 = 2 * R_1$, i.e., two APs are connected if circles of radius R_1 drawn around each AP intersect. We use a 170 node scenario where the nodes form 17 clusters (see Figure 3). Using the constrained clustering formulation taking into account the inter-AP connectivity, we obtain the output shown in Figure 3. We see that 6 APs are necessary for connecting the APs with one another and ensuring that all clusters are connected to at least one AP.

The results of the constrained clustering formulation with inter AP connectivity are compared with a Grid algorithm that performs an exhaustive search over the ground node area to find the minimum number of APs required to connect the different clusters and also have connectivity

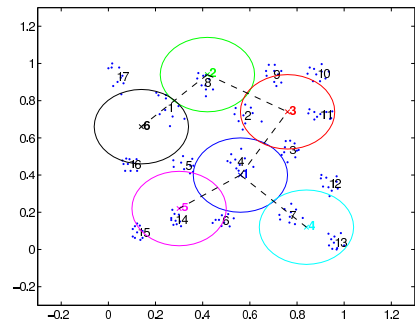


Fig. 4. Grid Algorithm: Number and location of APs.

among themselves. The Grid algorithm divides the area into a grid with a granularity of 0.02 and then performs an exhaustive search over all possible AP locations till it finds a solution. The algorithm starts with a single AP and then increments the number of APs until a solution is found. Obviously, this procedure is not scalable and can only be used in relatively small scenarios. The Grid algorithm when run with the same 170 node scenario also requires a minimum of 6 APs to ensure full connectivity both among the ground clusters and among each other. The output of the grid algorithm with the same R_0 , R_1 , and R_2 is shown in figure 4.

B. Capacity Constraints

To test the inclusion of capacity constraints, we used a simple scenario of 4 nodes arranged on the corners of a square with sides 0.35 forming 4 clusters (see figures 5 and 6). R_0 , R_1 , and R_2 are the same as in the previous section. C_1 (total capacity out of node 1 to all other nodes) and C_2 are set to 0.4 Mbps each. The corresponding capacity for nodes 3 and 4 is set to 0.8 Mbps. C_{max} is set to 1.0 Mbps. If capacity constraints are taken into account, a single AP can support both nodes 1 and 2 while nodes 3 and 4 need a separate AP each. Thus the minimum number of APs taking into account capacity constraints is 3 and this is shown in figure 6. The solution without taking into account capacity constraints requires 2 APs for full connectivity as seen in figure 5.

C. Single AP Survivable Network

We run the same 170 node example as used in section VI-A with the same values of R_0 ($= 0.1$), R_1 ($= 0.2$), and R_2 ($= 0.4$). For basic connectivity between the APs, we showed in section VI-A (figure 3) that 6 APs are necessary to connect all the ground clusters. Figure 7 shows the result of AP placement where each AP is constrained to connect to atleast two neighboring APs. We see that we need one additional AP, i.e., a total of 7 APs for this enhanced connectivity amongst APs. We can now trivially

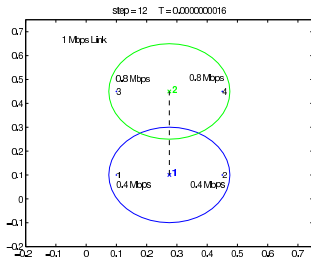


Fig. 5. Simple 4 Node Scenario: AP Placement for full connectivity without capacity constraints.

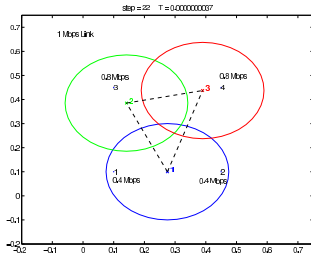


Fig. 6. Simple 4 Node Scenario: AP Placement for full connectivity with capacity constraints.

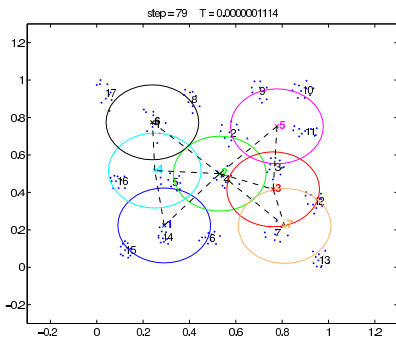


Fig. 7. AP Placement for enhanced connectivity (each AP is connected to 2 other APs): 7 APs needed.

make the network of APs in figure 7 single node survivable by placing an additional AP above each of the 7 APs. But this requires a total of 14 APs. Figure 8 is the result of the DA extension to make the network single node survivable. From the figure, we see that 10 APs are required (less than the trivial solution of 14 APs) to make the network single AP survivable. We see that each AP is connected to atleast 2 APs and each ground cluster is covered by atleast 2 APs.

VII. CONCLUSIONS

We have addressed the problem of providing full connectivity between disconnected ground clusters while at the same time satisfying required inter-cluster capacities by placing a minimum number of Aerial Platforms at appropriate locations. This problem is critical in ad hoc networks

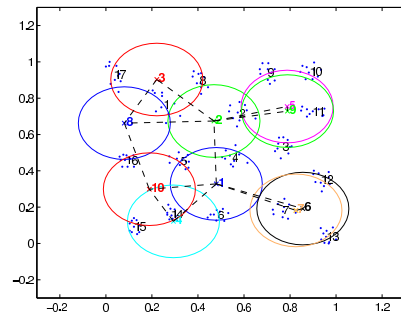


Fig. 8. AP Placement for single AP survivable network: 10 APs needed.

that need to have full connectivity and enough capacity between all ground clusters like in battlefield networks, rescue scenarios, etc. We use a constrained clustering formulation with complexity costs for solving this problem. The Deterministic Annealing clustering algorithm is used to avoid local minima and obtain near-optimal solutions. Our method of providing full connectivity is validated against an exhaustive search algorithm. We have also shown how to make the network of APs and ground clusters single AP survivable by extending the connectivity solution to have an AP connect to two neighboring APs and requiring that each ground cluster connect to atleast 2 APs.

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