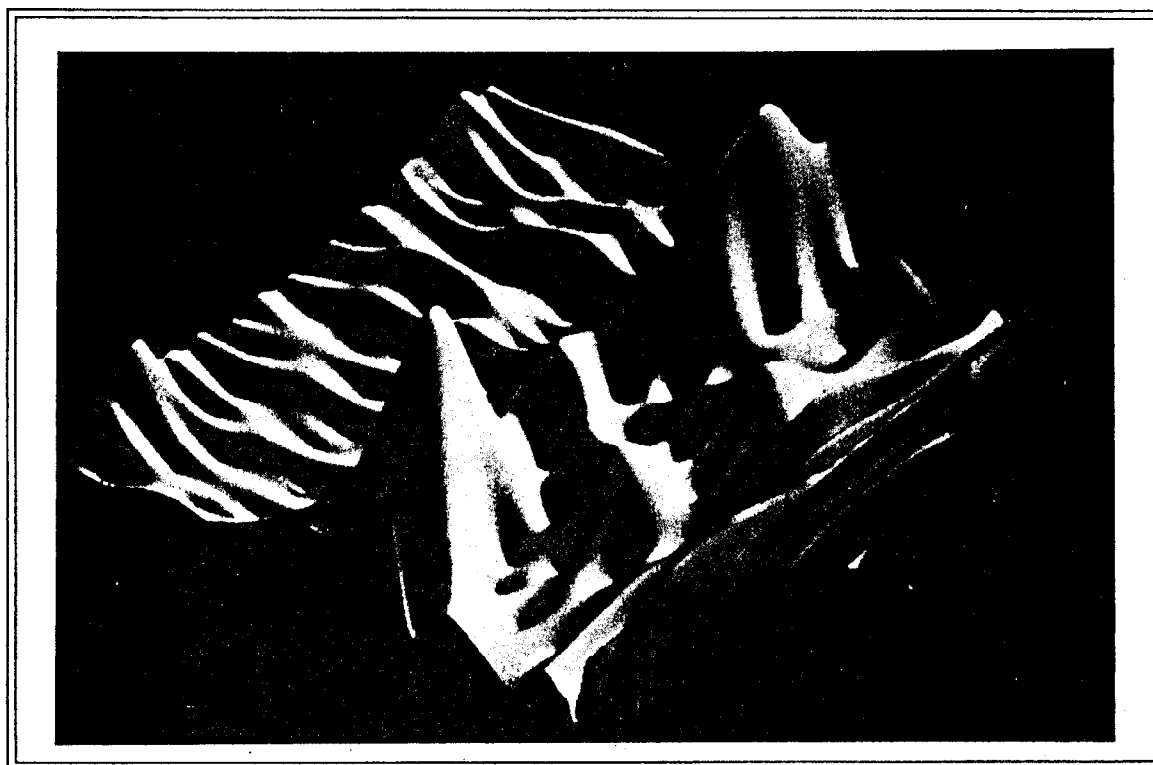


J.S. BARAS

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# An Analysis of Discrete Zero-Crossings and Maxima Wavelet Representations.

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## Summary

Recently, S. Mallat in series of papers [4, 2, 3] introduced zero-crossings and extrema of the wavelet transform as a multiscale edge representation. Due to the low complexity and flexibility in choosing the basic filter, this representation appears to be very promising for variety of applications. Moreover, Mallat and Zhong [3] show accurate numerical reconstruction results from maxima representation which seem to verify Marr's conjecture about possible completeness and stability of multiscale edge representation. From the theoretical point of view, there are still important open problems concerning the information they represent, e.g. stability, uniqueness, and structure of a reconstruction set.

Our aim is to analyze these questions in the practical case where data are discrete and finite. Our basic set-up is a bank of linear filters, which generalizes dyadic discrete wavelet transform, with a non-linear operation on the outputs (sampling extreme values or recording zero-crossings points). Our previous results have been reported in [1]. The main idea is to represent the reconstruction problem within a framework of linear programming.

Recently we have proven the following results:

1. Both maxima and zero-crossings discrete representations based on wavelet transform are in general non-unique. The exact statement is as follows: Given any maxima (zero crossings) discrete dyadic wavelet representation, generated by a discrete low pass filter  $H(\omega)$ . If  $H(\pi) = 0$ , then  $\forall N$  (number of samples) which is a multiplication of  $2^J$  and  $\forall J \geq 3$  (number of levels) there exists a sequence  $p = \{p(n)\}_{n=1}^N$  which has a non-unique maxima (zero-crossings) representation.

2. Both maxima and zero-crossings representation are stable in the sense, that for any bounded perturbation in the representation, the difference between any two functions such that one satisfies the nominal representation and the second corresponds to the perturbed one is bounded.

We conclude the paper with illustrations of reconstructions sets (sets of sequences having the same representation) and their sensitivity to different perturbations in the representation for some synthetic signals.

## References

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