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An Observer Design for Nonlinear Control Systems

M. R. James and J. S. Baras

Systems Research Center
University of Maryland
College Park, MD 20742, USA

Summary

We present an observer design for systems with controlled nonlinear dynamics and nonlinear observation. The design is a development of earlier work, which was motivated by nonlinear filtering asymptotics. The basic design requires that the initial conditions belong to a bounded region determined by the data and design parameters. However, for a certain class of systems, no such a priori knowledge is required. To illustrate the utility of our design, several examples are given.

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1 Introduction

In this paper we present a design for an observer for the nonlinear control system

$$\begin{aligned} \dot{x} &= f(x,u), \qquad x(0) = x_0, \\ y &= h(x) \end{aligned} \tag{1.1}$$

where $x \in I\!R^n$, $u \in I\!R^m$, $|u_i| \leq 1$ $i = 1,\ldots,m$ and $y \in I\!R^p$. The initial condition x_0 is unknown.

The *observer problem* consists of recursively computing an estimate $z(t)$ of $x(t)$ for which the error decays to zero as $t \to \infty$. That is, to design a system

$$\begin{aligned} \dot{m} &= F(m,u,y), \qquad m(0) = m_0, \\ z &= G(m) \end{aligned} \tag{1.2}$$

such that

$$\lim_{t\to\infty} |x(t) - z(t)| = 0 \tag{1.3}$$

for all x_0 in a suitable class $\mathcal{I}$. Here $\mathcal{I}$ represents a priori knowledge concerning the initial condition.

We prove the following result for our observer design: provided that we have some knowledge of x_0 (in the form $|x_0 - z_0| < \rho$, where z_0 is the initial estimate) and assuming that (1.1) satisfies a *detectability* condition, then the observer estimate $z(t)$ converges exponentially to the system trajectory $x(t)$ as $t \to \infty$ (Theorem 2.1). The radius of convergence ρ depends on the nonlinearities in the dynamics and observations as well as on certain design parameters. For a certain class of systems, no knowledge of x_0 is required (Corollary 2.1).

Our design is a development of the design given in Baras, Bensoussan and James [1], which treats systems with uncontrolled nonlinear dynamics and linear observations. The main contributions here are the results for nonlinear observations and controlled dynamics. We remark that these designs do not involve coordinate transformations, canonical forms, local linearization, etc, and seem robust when compared with other designs. However, the designs do involve solving Riccati equations and computing certain matrices and constants.

In Section 2 we give the observer design and state the main convergence results. The design involves Riccati differential equations with time-varying coefficients, and in Section 3 we obtain bounds on the solutions of these equations under certain detectability and rank conditions. These bounds are used in Section 4 to prove the convergence result. Finally in Section 5 we give several examples.

2 Observer Design

We assume that f, h are smooth with bounded derivatives of orders 1 and 2. Let $N \in L(I\!R^n, I\!R^n)$, $R \in L(I\!R^p, I\!R^p)$ and assume rank $N = n$ and $R > 0$. Assume $t \mapsto u(t)$ is continuous.

Write $A(x,u) = Df(x,u)$, $H(x) = R^{-1}Dh(x)$, where D denotes gradient in the x variable. Set

$$\|A\| = \sup\{\|A(x,u)\| : x \in I\!R^n, |u_i| \leq 1\}$$

and similarly define $\|H\|$, and so on.

Consider the coupled system

$$\begin{aligned} \dot{m}(t) &= f(m(t),u(t)) + Q(t)H(m(t))'R^{-1}(y(t) - h(m(t))) \qquad (2.1)\\ m(0) &= m_0 \\ \dot{Q}(t) &= A(m(t),u(t))Q(t) + Q(t)A(m(t),u(t))' \\ &\quad -Q(t)H(m(t))'H(m(t))Q(t) + NN' \qquad (2.2)\\ Q(0) &= Q_0 > 0. \end{aligned}$$

This is our observer for (1.1). It is essentially a modification of the deterministic or minimum energy estimator, as discussed in Baras, Bensoussan and James [1]. Note in particular that the Riccati differential equation (2.2) depends on the control. This is not necessary when $f(x,u) = f(x) + Bu$: set $A(x) = Df(x)$.

Refering to Section 3, we will assume that the pair $(H(x), A(x,u))$ is *uniformly detectable*. Since N has rank n and $\|A\| < \infty$, the pair $(A(x,u), N)$ is *uniformly stabilisable* (refer to Section 3), and $NN' \geq r_0 I$ for some $r_0 > 0$. Let $P_0 = Q_0^{-1}$, $P(t) = Q(t)^{-1}$, and let p, q be the bounds for $\|P(t)\|$, $\|Q(t)\|$ (given in Section 3).

Regard A_0, N, R as design parameters. Define $\rho = \rho(Q_0, N, R)$ by

$$\rho = \frac{r_0}{q^2 \| P_0^{1/2} \|} \left(\sqrt{p} \| D^2 f \| + \sqrt{q} \| R^{-1} \|^2 \| Dh \| \| D^2 h \| \right)^{-1} \tag{2.3}$$

Our main theorem is the following convergence result, similar to Theorem 8 in [1].

Theorem 2.1 *Assume there exists Q_0, N, R such that*

$$|x_0 - m_0| < \rho(Q_0, N, R) \tag{2.4}$$

Then the system (2.1), (2.2) is an observer for the nonlinear control system (1.1) provided that $(H(x), A(x,u))$ is uniformly detectable and the above assumptions hold. That is, there exists contants $K > 0, \gamma > 0$ such that

$$|x(t) - m(t)| \leq K |x_0 - m_0| e^{-\gamma t} \tag{2.5}$$

for all $t \geq 0$.

Remark There is a trade-off between the decay rate $\gamma = \gamma(Q_0, N, R)$ and the radius of convergence ρ. The designer can optimize his choice of γ, ρ by varying the design parameters. ///

By using different estimates for the nonlinearities, we obtain an observer for (1.1) without any contraints on the initial conditions x_0, m_0 for a class of systems. Included in this class are systems for which $A(x,u)$ is uniformly negative definite (see the example in Section 5.2).

Define $\delta = \delta(Q_0, N, R)$ by

$$\delta = \frac{r_0}{q^2} - 4p\|Df\| - 4\|R^{-1}\|^2 \|Dh\|^2. \tag{2.6}$$

If $D^2 f$ or $D^2 h$ is zero, we omit the corresponding term from (2.6).

Corollary 2.1 *Assume there exist Q_0, N, R such that*

$$0 < \delta(Q_0, N, R). \tag{2.7}$$

Then the system (2.1), (2.2) is an observer for the control system (1.1) provided that $(H(x), A(x,u))$ is uniformly detectable and the above assumptions hold. That is, there exists contants $K > 0, \gamma > 0$ such that

$$|x(t) - m(t)| \leq K |x_0 - m_0| e^{-\gamma t} \tag{2.8}$$

for all $t \geq 0$ and all $x_0, m_0 \in \mathbb{R}^n$.

Remark Our design can readily be extended to time varying systems, provided one extends the definition of uniform detectability. ///

3 Riccati Equations

Write $X = \mathbb{R}^n \times [-1,1]^m$ and $\xi = (x,u) \in X$. If $t \mapsto \xi_t = (x_t, u_t)$ is a continuous curve, we write $A_t = A(\xi_t) = A(x_t, u_t)$, etc.

Consider the Riccati differential equations

$$\begin{aligned} \dot{Q}_t &= A_t Q_t + Q_t A_t' - Q_t H_t' H_t Q_t + NN' && (3.1) \\ \dot{P}_t &= -P_t A_t - A_t' P_t - P_t N N' P_t + H_t' H_t && (3.2) \\ Q_0 &= P_0^{-1} > 0 \end{aligned}$$

Existence and uniqueness for (3.1), (3.2) are standard, and note that $P_t = Q_t^{-1}$.

3.1 Uniform Detectability and Stabilisability

In this section we present sufficient conditions that ensure boundedness of the solutions of the Riccati equations (3.1), (3.2). The bound for $\| Q_t \|$ requires a detectability condition which we now define.

Definition *The pair of matrices $(H(\xi), A(\xi))$ is uniformly detectable if there exist a constant $\alpha_0 > 0$ and a bounded continuous matrix valued function $\Lambda(\xi)$ such that*

$$\eta' (A(\xi) + \Lambda(\xi) H(\xi)) \eta \leq -\alpha_0 \mid \eta \mid^2 \tag{3.3}$$

for all $\eta \in \mathbb{R}^n$, $\xi \in X$.

This condition is similar to the well known detectability condition for linear time-invariant systems. The pair of matrices (C, A) is *detectable* if there exists a matrix Λ such that the eigenvalues of $A + \Lambda C$ have strictly negative real parts; uniform detectability implies detectability, but not conversely. A disadvantage of this condition is that it is in general difficult to check, and $\Lambda(x,u)$ may be hard to compute. No simple rank-type condition exists to date. In the case that $H(x)$ is *uniformly of full rank*, that is,

$$H(x)'H(x) \geq s_0 I, \tag{3.4}$$

for some $s_0 > 0$, it is possible to bound $\| Q_t \|$ without using (3.3).

To obtain a uniform bound for $\| P_t \|$, we assume that $\mathrm{rank}N = n$ and use the following *uniform stabilisation* result, based on Kalman [2]. Let $\Phi_F(t,t_0)$ denote the fundamental transition matrix corresponding to a time varying matrix F_t. Recall $NN' \geq r_0 I$.

Lemma 3.1 *Assume rankN = n. Consider the control system*

$$\dot{z}_t = -A_t z_t - N u_t, \quad z(0) = z, \tag{3.5}$$

where $A_t = A(\xi_t)$ for some curve $t \mapsto \xi_t$. Then there exists a feedback control $u_t^o = \Gamma_t z_t$ such that

$$\| \Phi_{\tilde{A}}(t,t_0) \| \leq \sqrt{\frac{\beta_1}{\beta_0}} \exp\left(-\frac{1}{2\beta_1}(t-t_0)\right), \tag{3.6}$$

for $t \geq t_0 \geq 0$, where $\tilde{A}_t = -A_t - N\Gamma_t$ and for any $\sigma > 0$,

$$\beta_0(\sigma) = \sigma e^{-2\sigma\|A\|}\left(1 + \sigma^2 e^{2\sigma\|A\|} \| N \|^2\right)^{-1},$$

$$\beta_1(\sigma) = \sigma e^{4\sigma\|A\|}\left(1 + \frac{\| N \|^2}{r_0 \sigma}\right),$$

$$\| \Gamma_t \| \leq \| N \| \beta_1(\sigma) \equiv \| \Gamma \|.$$

Note that the bounds are independent of the curves $t \mapsto \xi_t$.

3.2 Bounds

Theorem 3.1 *Assume that $\xi \mapsto A(\xi), H(\xi)$ are continuous and bounded, $(H(\xi), A(\xi))$ is uniformly detectable, and $\mathrm{rank}N = n$. Then we have*

$$\|Q_T\| \leq \left(\|Q_0\| + \frac{\|N\|^2 + \|\Lambda\|^2}{2\alpha_0}\right) \equiv q < \infty, \tag{3.7}$$

$$\|P_T\| \leq \left(\frac{\beta_0}{\beta_1}\|P_0\| + \frac{\|H\|^2 + \|\Gamma\|^2}{2\beta_0}\right) \equiv p < \infty. \tag{3.8}$$

These bounds are independent of $T > 0$.

Note The bound q depends on the choice of Λ, while p depends on σ. To obtain the best bound, one can optimise over these parameters. For linear time-invariant detectable systems, one can also

obtain a bound for $\| Q_t \|$. ///

Proof: We modify an argument in [1]. To prove (3.7), consider the following linear optimal control problem with time-varying coefficients:

$$-\dot{\eta}_t = A_t'\eta_t + H_t'v_t, \qquad \eta_T = h, \tag{3.9}$$

where $h \in I\!R^n$ is given and v is the control. The cost functional is

$$J_1(v,T) = \eta_0' Q_0 \eta_0 + \int_0^T (v_t'v_t + \eta_t' N N' \eta_t) dt. \tag{3.10}$$

Define a value function

$$V_1(h,T) = \inf\{J_1(v,T) : \eta_T = h\}$$

The Hamilton-Jacobi-Bellman (HJB) equation is

$$\frac{\partial}{\partial T}V_1 + \max_v[D_\eta V_1(-A_t' - H_t'v) - v^2 - \eta' N N_\eta'] = 0$$

Let Q_t be the solution of (3.1). Then

$$V(\eta,t) = \eta' Q_t \eta$$

is the unique (viscosity) solution of (3.2) with $V(\eta,0) = \eta' Q_0 \eta$.

Consider the (suboptimal) feedback control law

$$v(t) = \Lambda_t' \eta_t.$$

Then by (3.9),

$$-\dot{\eta}_t = (A_t' + H_t'\Lambda_t')\eta_t, \quad \eta_T = h. \tag{3.11}$$

Then we have

$$V(\eta,T) = h'Q_T h \leq \eta_0 Q_0 \eta_0 + \int_0^T \eta_t'(NN' + \Lambda_t\Lambda_t')\eta_t dt \tag{3.12}$$

Now using (3.11),

$$|\eta_0|^2 - 2\int_0^T \eta_t'(A_t' + H_t'\Lambda_t')\eta_t dt = |h|^2$$

Hence using uniform detectability (3.3), $|\eta_0|^2 \leq |h|^2$ and

$$\int_0^T |\eta_t|^2 dt \leq \frac{|h|^2}{2\alpha_0}.$$

Combining this with (3.12) we obtain

$$h'Q_T h \leq h'\left(\|Q_0\| + \frac{\|N\|^2 + \|\Lambda\|^2}{2\alpha_0}\right)h$$

which proves (3.7).

Similarly, to prove (3.8), consider the optimal control problem

$$\dot{\lambda}_t = A_t \lambda_t + N v_t, \quad \lambda_T = h \tag{3.13}$$

with cost

$$J_2(v,T) = \lambda_0' P_0 \lambda + \int_0^T (v_t' v_t + \lambda_t' H_t' H_t \lambda_t) dt.$$

One uses the control $v(t) = \Gamma_t \lambda_t$ from Lemma 3.1. The details will be omitted. ∎

Corollary 3.1 *Assume that $\xi \mapsto A(\xi)$, $H(\xi)$ are bounded and continuous, and that $H(\xi)$ is uniformly of full rank. Then*

$$\| Q_T \| \leq \left(\frac{\alpha_0}{\alpha_1} \| Q_0 \| + \frac{\| N \|^2 + \| \Lambda \|^2}{2\alpha_0} \right) \equiv q \leq \infty, \tag{3.14}$$

for all $T > 0$, where for any $\tau > 0$,

$$\alpha_0(\tau) = \tau e^{-2\tau\|A\|} \left(1 + \tau^2 e^{2\tau\|A\|} \| N \|^2\right)^{-1},$$

$$\alpha_1(\tau) = \tau e^{4\tau\|A\|} \left(1 + \frac{\| N \|^2}{r_0 \tau}\right),$$

$$\| \Lambda_t \| \leq \| H \| \, \alpha_1(\tau) \equiv \| \Gamma \| .$$

4 Asymptotic Convergence

Using the bounds (3.7), (3.8) we prove Theorem 2.1, and Corollary 2.1.

Proof of Theorem 2.1: The error $e(t) = x(t) - m(t)$ satisfies

$$\begin{aligned} \dot{e}(t) &= f(x(t),u(t)) - f(m(t),u(t)) - Q(t)H(m_t)'R^{-1}(y(t) - h(m(t))) \\ &= [A(m(t),u(t)) - Q(t)H(m(t))'H(m(t))]e(t) \\ &\quad + [f(x(t),u(t)) - f(m(t),u(t)) - Df(m(t),u(t))e(t)] \\ &\quad - Q(t)H(m(t))'R^{-1}[h(x(t)) - h(m(t)) - Dh(m(t))e(t)] \end{aligned}$$

Therefore using the Riccati equation (3.2) for $P(t)$,

$$\begin{aligned} \frac{d}{dt} e(t)' P(t) e(t) &= -e(t)' P(t) N N' P(t) e(t) - e(t)' H(m(t))' H(m(t)) e(t) \\ &\quad + 2e(t)' P(t) \int_0^1 \int_0^1 r D^2 f(m(t) + rse(t), u(t)) e(t)^2 dr ds \\ &\quad - 2e(t)' H(m(t))' R^{-1} \int_0^1 \int_0^1 r D^2 h(m(t) + rse(t)) e(t)^2 dr ds \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} |P(t)^{\frac{1}{2}} e(t)|^2 &\leq e(t)' \left(-r_0/q^2 + |P(t)^{\frac{1}{2}} e(t)| [\sqrt{p} \| D^2 f \| \right. \\ &\quad \left. + \sqrt{q} \| R^{-1} \|^2 \| Dh \| \| D^2 h \|] \right) e(t) \end{aligned} \tag{4.1}$$

Let $C = (\sqrt{p}\|D^2 f\| + \sqrt{q}\|R^{-1}\|^2 \|Dh\| \|D^2 h\|)$. By hypothesis (2.4) we have

$$-\frac{r_0}{q^2} + |P_0^{\frac{1}{2}} e_0| C < 0.$$

Since $P(t)^{\frac{1}{2}} e(t)$ is continuous, there is an interval $[0, t_0)$ such that

$$-\frac{r_0}{q^2} + |P(t)^{\frac{1}{2}} e(t)| C < 0 \quad \text{on} \quad [0, t_0).$$

But then (4.1) implies

$$\frac{d}{dt} |P(t)^{\frac{1}{2}} e(t)|^2 < 0 \quad \text{on} \quad [0, t_0),$$

and thus

$$|P(t)^{\frac{1}{2}} e(t)| \leq |P_0^{\frac{1}{2}} e_0|$$

for $t \in [0, t_0)$. By continuity this inequality holds for $t \in [0, t_0]$. Hence we can proceed from t_0 on. Thus there exists $\delta > 0$ such that

$$|P(t)^{\frac{1}{2}} e(t)| \leq \frac{1}{C} \left(\frac{r_0}{q^2} - \delta \right)$$

for all $t \geq 0$. So (4.1) implies

$$\frac{d}{dt} |P(t)^{\frac{1}{2}} e(t)|^2 \leq -\delta |e(t)|^2.$$

But from (3.8)

$$e(t)' P(t) e(t) \leq \| P(t) \| \, |e(t)|^2 \leq p |e_t|^2,$$

so that

$$\frac{d}{dt} e(t)' P(t) e(t) \leq -\frac{\delta}{p} e(t)' P(t) e(t),$$

which implies

$$e(t)' P(t) e(t) \leq e(0)' P_0 e(0) e^{-\frac{\delta}{p} t}, \quad t \geq 0.$$

Therefore, using (3.7), we have

$$\begin{aligned} |e(t)|^2 &\leq q \, e(t)' P(t) e(t) \\ &\leq q \, e(0)' P_0 e(0) e^{-\frac{\delta}{p} t}, \quad t \geq 0, \end{aligned}$$

which implies (2.5). ∎

Proof of Corollary 2.1: We have

$$\begin{aligned}
\frac{d}{dt}e(t)'P(t)e(t) &= -e(t)'P(t)NN'P(t)e(t) - e(t)'H(m(t))'H(m(t))e(t) \\
&\quad +2e(t)'P(t)(f(x(t),u(t)) - f(m(t),u(t)) - Df(m(t),u(t)))e(t)) \\
&\quad -2e(t)'(R^{-1}Dh(m(t))'R^{-1}(h(x(t)) - h(m(t)) - Dh(m(t)e(t)) \\
&\leq \left(-\frac{r_0}{q^2} + 4p\|Df\| + 4\|R^{-1}\|^2\,\|Dh\|^2\right)|e(t)|^2
\end{aligned}$$

By assumption (2.7) there is a $\delta > 0$ such that

$$-\frac{r_0}{q^2} + 4p\|Df\| + 4\|R^{-1}\|^2\,\|Dh\|^2 = -\delta < 0.$$

Therefore

$$\frac{d}{dt}|P(t)^{\frac{1}{2}}e(t)|^2 \leq -\delta|e(t)|^2$$

for all $t \geq 0$ and all $e_0 \in I\!R^n$. This implies (2.8). ∎

5 Examples

5.1 Bilinear Dynamics, Linear Observation

Consider the general bilinear system

$$\begin{aligned}
\dot{x} &= \left(A + \sum_{i=1}^{m} u_i B_i\right)x, \qquad x(0) = x_0, \qquad (5.1)\\
y &= Cx.
\end{aligned}$$

We assume $|u_i| \leq 1$, $p = 1$, and here $\xi = u \in [-1,1]^m = X$. Write

$$A(u) = A + \sum_{i=1}^{m} u_i B_i.$$

Define, for $\tau > 0$, the observability grammian

$$\mathcal{O}(t_0, t_0+\tau) = \int_{t_0}^{t_0+\tau} \Phi_{A'}(t_0,t)C'C\Phi'_{A'}(t_0,t)dt,$$

where $A_t = A(u(t))$. Assume that (5.1) is *uniformly observable* in the sense that there exists $\tau > 0$ such that for all $t_0 \geq 0$

$$\gamma_0(\tau)I \leq \mathcal{O}(t_0, t_0+\tau) \leq \gamma_1(\tau)I$$

for constants $\gamma_0(\tau)$, $\gamma_1(\tau) > 0$, independent of the control. The we can bound $\| Q_t \|$ as in Corollary 3.1.

Then the following system is an observer for (5.1), with no contraints on the initial conditions.

$$\begin{aligned}
\dot{m}(t) &= A(u(t))m(t) + Q(t)C'(y(t) - Cm(t)),\ m(0) = m_0, \qquad (5.2)\\
\dot{Q}(t) &= A(u(t))Q(t) + Q(t)A(m(t))' - Q(t)C'CQ(t) + I,\ \ Q_0 = I.
\end{aligned}$$

For simplicity we have taken Q_0, N, R to be identity matrices. To improve the decay rate γ, one could try other values for Q_0, N, R.

Compare this design with the design for linear time-varying systems in Willems and Mitter [5], and O' Reilly [4].

5.2 Linear Dynamics, Nonlinear Observations

Consider the system

$$\begin{aligned}
\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}u, \qquad (5.3)\\
y &= \sin x_1.
\end{aligned}$$

This system is controllable and observable. However, the pair of matrices $(Dh(x), A)$ is not observable for $x_1 = k\frac{\pi}{2}$, where k is an odd integer. The system has eigenvalues $-1, -2$ and A is symmetric, hence $(Dh(x), A)$ is automatically uniformly detectable, with $\alpha_0 = 1$, $\Lambda(x) \equiv 0$. Let $R = rI$, $N = \sqrt{r_0}I$, $Q_0 = \gamma^2 I$. Here, $H(x) = \frac{1}{r}(\cos x_1, 0)$. Now

$$\delta = r_0(\gamma^2 + r_0/2)^{-2} - 4r^{-2}.$$

Set $r = 3$, $r_0 = 0.2$, $\gamma = 0.1$. Then $\delta = 7.82$.

The observer for (5.3) is

$$\begin{aligned}
\dot{m}(t) &= Am(t) + Bu(t) + \frac{1}{3}Q(t)H(m(t))'(y(t) - \sin m_1(t)), \qquad (5.4)\\
\dot{Q}(t) &= AQ(t) + Q(t)A' - Q(t)H(m(t))'H(m(t))Q(t) + 0.2I.
\end{aligned}$$

By Corollary 2.1, $m(t)$ converges exponentially to $x(t)$ for all $x_0, m_0 \in I\!R^n$.

References

[1] J.S. Baras, A. Bensoussan and M.R. James, *Dynamic Observers as Asymptotic Limits of Recursive Filters: Special Cases*, Technical Report SRC-TR-86-79, Systems Research Center, University of Maryland, December 1986. To appear, *SIAM J. Applied Math.*

[2] R. E. Kalman, *Contributions to the Theory of Optimal Control*, Bol. Soc. Mat. Mex., 1960, pp102-119.

[3] S. R. Kuo, D. L. Elliott, and T. J. Tarn, *Exponential Observers for Nonlinear Dynamic Systems*, Inform. Cont. 29 (1975) 204-216.

[4] J. O'Reilly, *Observers for Linear Systems*, Academic Press, London, 1983.

[5] J. C. Willems and S. K. Mitter, *Controllability, Observability, Pole Allocation, and State Reconstruction*, IEEE Trans. Aut. Cont. AC-16(6), (1971) 582-595.