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Decomposition and Decentralized Control System Design: A Review of Frequency Domain Methods

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ABSTRACT

We provide a short review of the literature on design of decentralized control based on weak coupling measures for transfer function models. The notion of diagonal dominance for transfer functions has been suggested as a measure of weak coupling for system decomposition. Various generalizations of this notion for partitioned transfer function matrices are discussed as they pertain to design of decentralized control.

Such weak coupling techniques permit a decentralized design procedure based on an approximate decoupled model. The accuracy of these decoupled approximations for control design is highlighted.

1.0 Introduction

Various methods for decentralized control design have been suggested based on an assumption of subsystem "weak coupling" of the input-output system response which permit an approximate decoupled system model to be used for design. For linear design problems, decomposition of system frequency domain models can provide a natural way to construct decentralized control. In this paper we will review developments in the theory and application of weak coupling measures in terms of transfer function models and techniques for decentralized control. We remark that in contrast to use of diagonal dominance and other weak coupling techniques for state space models these methods have received rather limited attention in the control literature in recent years. Interest in such methods may have suffered to some extent from a lack of understanding of the physical significance of weak coupling evidenced in the transfer function. We refer the reader to the other papers in this session for details of several applications. We hope that by examining the issues relevant to a series of applications from various problem areas that insight can be gained as to the efficacy of these methods and the significance of the various notions of weak coupling of the system input-output response.

2.0 Decomposition of Models and Weak Coupling

The utility of the state space framework for modeling follows from the natural way in which time domain models are obtained for large systems, e.g., large network problems. This is in part due to the inherent flexibility of state space modeling via isomorphism of the state space. Thus a state space model can usually be obtained in which particular system parameters can be made to appear in the elements of the state space equation (matrix) operators in a simple (e.g. linear) manner. This fact makes it attractive to conduct perturbation analysis of various "weak coupling" parameters in the state space model. Such a notion of weak coupling is highlighted in the comprehensive survey of Sandell, et al [24].

The use of diagonal dominance and the related concepts of M-matrices has been applied to state space modeling and stability analysis for large scale systems by various researchers [14,25]. In these studies the stability of solutions to a set of coupled differential equations was studied by examining the dominance properties of the matrix "generator", A, for the state space model, $\dot{x}=Ax$. A simple observation of Willems [28] is that for $A=[a_{ij}]$ diagonally dominant with $a_{ii}<0$ its eigenvalues can be shown to have strict negative real parts. The resulting system is of course stable. Various constructive techniques have been developed for obtaining Lyapunov functions for systems with such a composite "weakly" coupled structure [14,25]. In this context the association between diagonal dominance, spectral estimates of the Gerschgorin type, and the theory of M-matrices has been exploited. Siljak employed these notions and the idea of a vector Lyapunov function to obtain a method for stability analysis and design of decentralized control with state feedback. Various enhancements of such methods including generalizations of diagonal dominance and extensions to partitioned matrices have been applied to this problem by Siljak [25].

The suggestion that feedback control could be used to enhance weak coupling has been studied in the context of the decoupling control problem. The objective of decoupling being naturally interpreted as a requirement to make the resulting closed loop system have impulse response matrix which is diagonal (or nearly so). In the frequency domain a simple way to view weak coupling is via the notion of a diagonally dominant transfer function matrix. The introduction of diagonal dominance for transfer functions in the control theoretic literature was due to Rosenbrock [23]. Various interaction indices for composite systems based on time domain models have also been proposed, but, by and large, the use of diagonal dominance (and related notions) have dominated studies based on input-output models. As a result the question is often asked "what does diagonal dominance of the transfer function mean in the state space model?" This question remains largely unanswered although an interaction index proposed by Aplevich[1] was shown to satisfy particular bounds for stable systems with diagonally dominant transfer functions. This result was shown in the context of some work by Hutcheson [11] on design of decoupling control. It is apparent that there is no clear relationship between weak coupling for state space models and for input-output models.

2.1 Diagonal Dominance Methods: Refinements and Extensions

The application of diagonal dominance to control system design was popularized by Rosenbrock in the development of the Inverse Nyquist Array (INA) method of multivariable, loop-at-a-time design [23]. The success of this method for decentralizing the design

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process followed from two significant aspects of diagonally dominant transfer functions. First, the fact that diagonally dominant matrices are nonsingular, is exploited to provide a stability test for decentralized feedback. Second, for design, one can use the "decoupled" model with the accuracy of this approximation given in terms of the amount of dominance.

The stability result follows from an application of a Nyquist test for square transfer functions. We take D to be the usual closed contour in the complex plane consisting of a relatively large portion of the imaginary axis (with possible indentations for imaginary poles) and a semicircular arc in the right half plane.

Throughout this paper we will consider the multivariable feedback equations:

$$y(s) = G(s)\, e(s) \qquad (2.1)$$
$$e(s) = u(s) - F(s)\, y(s)$$

with y(s) a p-vector and e(s),y(s) m-vectros, which lead to the closed loop transfer function from u(s) to y(s);

$$H(G,F,s) = G(s)[I_m + FG(s)]^{-1}. \qquad (2.2)$$

Let the matrix return difference be

$$R(s) = I_m + F(s)G(s). \qquad (2.3)$$

With respect to a given nXn partition of $R=[R_{ij}]$ we designate the splitting of the matrix using the notation; $R = R^D + R^C$ [1] with

$$R^D = \text{diag}\ \{ R_{11}, R_{22}, \ldots, R_{nn} \}\ .$$

<u>2.1 Theorem (decentralized stabilization)</u>: Under the following assumptions:

(i) $F^D G^D$ and FG have the same number of poles in the closed right half plane (CRP)

(ii) $H(G^D,F^D)$ is an asymptotically stable transfer function

(iii) $\det[R^D + \theta R^C] \neq 0$ for all $s \in D$, and for all $\theta \in [0,1]$,

Then the closed loop transfer function H(G,F) is also asymptotically stable.

<u>Remark</u>: Theorem 2.1 has been stated (and proven) explicitly in [3,18]. The result has been proven implicitly in [12,17].

The objective of various weak coupling results (which have been proposed as extensions of INA methods) for stabilizing decentralized control is to provide sufficient conditions for assumption (iii) in theorem 2.1. This amounts to a homotopical equivalence between a pair of Nyquist curves; viz. one for the decoupled approximate system and one for the true coupled system. Rosenbrock used the trivial mXm partitioning and the classical Gerschgorin results to guarantee assumption (iii). We view assumption (iii) as a kind of weak coupling result which is appropriate to stability considerations.

It will be convenient to define diagonal dominance in the following manner.

1 From now on we will supress the argument s in the statement of transfer functions whenever convenient.

<u>2.2 Definition</u>: Given an mXm matrix, Z(s), rational in s we construct a pair of mXm test matrices $B=[b_{ij}]$, $C=[c_{ij}]$ as follows:

$$B = \text{diag}\ \{ |z_{11}|, |z_{22}|, \ldots, |z_{mm}| \} \qquad (2.5)$$

$$c_{ij} = \begin{cases} 0, & \text{for } i=j \\ |z_{ij}|, & \text{for } i\neq j\ . \end{cases} \qquad (2.6)$$

Let the real m-vector $\underline{1} = (1,\ldots,1)^t$. Then we say that the matrix Z is <u>diagonally dominant</u> by rows if:

$$[I_m - B^{-1}C]\ \underline{1} > 0 \quad \text{(element wise)} \qquad (2.7)$$

and by columns if:

$$[I_m - CB^{-1}]\ \underline{1} > 0\ . \qquad (2.8)$$

The major problem with the INA method for design of general multivariable control systems is that often G(s) is not diagonally dominant. In this case, the method as developed by Rosenbrock, requires the construction of pre- (and/or post-) series compensators such that Q(s) = L(s)G(s)K(s) can be made diagonally dominant. Available techniques for the synthesis of K(s) and L(s) which represent relatively low order, realizable, and stable multivariable plants are ad hoc at best [23].

We take the viewpoint that synthesis of K(s) and L(s) may not be of fundamental interest for two reasons. There is an available theory for the construction of optimal estimates of the class of transfer functions which are weakly coupled in a more general sense (in that they satisfy (iii) of theorem 2.1) and which may provide the necessary estimates when the plant is not diagonally dominant in the usual sense [3-6,12,15-17]. Second, for large scale systems, we are interested in identifying the structural aspects of the plant which admit a decentralized control solution embodying more general partitioning of the information pattern imposed on the controller. (Note that in general use of pre- (and/or post-) compensation is not consistent with constraints on the information pattern for decentralized control.)

2.1.1 Optimally Sharp Stability Estimates for Weakly Coupled Systems

It can be readily seen that, with respect to the usual definition of diagonal dominance of a transfer function, the weak coupling measure does not include an obvious class of weakly coupled systems; viz., systems with transfer function matrix which is upper (resp. lower) triangular but <u>not</u> diagonally dominant. Limebeer [12], recognizing this deficiency suggested a refined definition of dominance. Independently, Nwokah [15-17] applied the theory of M-matrices to the test matrix $[I_m - B^{-1}C]$ appearing in (7) to develop a sharper stability estimate. Both efforts led to essentially the same notion of generalized diagonal dominance (GDD) which we summarize in the following theorem.

<u>2.4 Theorem (GDD)</u>: Given an mXm rational matrix Z we construct the test matrices B and C as in (2.5)-(2.6). Then Z is <u>generalized diagonally dominant</u> (GDD) if any of the following equivalent conditions hold:

(i) there exists a real m-vector x>0 such that

$$[I_m - B^{-1}C]\ x > 0$$

(ii) there exists an m-vector y>0 such that

$$[I_m - CB^{-1}]\ y > 0$$

(iii) the test matrix $[I_m - CB^{-1}]$ is an M-matrix [2]

(iv) if the test matrix $[I_m + CB^{-1}]$ (a strictly nonnegative matrix) has a dominant Perron eigenvalue $\lambda_{PF}>2$. [3]

If Z is GDD then Z is nonsingular on D.

The work of Fiedler and Ptak [10] provides the basis for the consistency of this definition of GDD. More significantly, Varga [26] has shown that the resulting Gerschgorin disks i=1,...,m

$$G_i(x) = \{ w \in C: |w - z_{ii}| \le \sum_{j=1}^{m} |a_{ij}| \frac{x_j}{x_i} \}$$

are optimally sharp with respect to the information in the test matrices B and C. To make this precise consider the class of all mXm matrices which generate the same m Gerschgorin sets $G_i(x)$. Varga showed that the boundary of the union of the m $G_i(x)$ is composed of spectra from matrices in this class. Thus for the information contained in the test matrices B and C the scaling x>0 (or equivalently y>0) is optimal.

2.1.2 Block Diagonal Dominance and Decentralized Stabilization

In references [9-10,27-28] results are provided which generalize the Gershgorin theories to the case of partitioned matrices in several ways. In [3] these ideas are applied to the problem of decentralized feedback control by appropriately generalizing the Nyquist array ideas of Rosenbrock. Unlike in Rosenbrock's loop-at-a-time INA method, the resulting design approach involves a sequence of multivariable designs.

Let $Z=[Z_{ij}]$ be an nXn complex matrix partitioned in mXm submatrices, where Z_{ij} is $k_i X k_j$; and $\sum_{j=1}^{m} k_j=n$. In [9] block diagonal dominance (BDD) is defined analogously with the previous definition except for the construction of the test matrices B and C. Given an nXn matrix, Z(s), rational in s, and partitioned as above we construct a pair of mXm test matrices $B=[b_{ij}]$, $C=[c_{ij}]$ as follows:

$$B = \mathrm{diag}\{ ||Z_{11}^{-1}||, ||Z_{22}^{-1}||, \ldots, ||Z_{mm}^{-1}|| \} \quad (2.9)$$

$$c_{ij} = \begin{cases} 0, & \text{for } i=j \\ ||Z_{ij}||, & \text{for } i \ne j \end{cases} \quad (2.10)$$

Then [3] we say that the rational matrix Z is block diagonally dominant (BDD) if either (2.7) or (2.8) is satisfied (with m replaced by n). We remark that in contrast to the previous definition of diagonal dominance there is considerable flexibility in this definition in terms of the choice of norms on individual subspaces and the choice of partitioning. Indeed, considerations for the optimality of the partitioned stability estimates are much more delicate than the situation discussed in section 2.1.1 [27].

2 Recall that a matrix with nonpositive off-diagonal elements is an M-matrix if all its principal minors are strictly positive.

3 The Perron Frobenius root of an irreducible nonnegative matrix is the unique eigenvalue of maximum modulus.

Limebeer suggested the notion of generalized block diagonal dominance (GBDD) by applying these same test matrices to the four conditions of theorem 2.4. Limebeer preferred (for obvious computational reasons) the evaluation of GBDD based on condition (iv) of this theorem since this reduces to computing an eigenvalue of maximum modulus. In the case that the matrix $[I_n + CB^{-1}]$ is irreducible (in the sense of the Perron-Frobenius theory of nonnegative matrices) this eigenvalue is unique. Moreover, the associated eigenvector gives the optimal scaling for GBDD used in the vectors x and y of theorem 2.4 (where in this case x=y).

More recently, Ohta et al [18] have applied a result of Okuguchi [20] on partitioned matrices. They suggest to form instead the test matrix $\hat{B} = (\hat{b}_{ij})$ where

$$\hat{b}_{ij} = ||Y_{ij}|| \quad (2.11)$$

with $Y = Z^C (Z^D)^{-1}$ partitioned conformally with Z. Here Z is split as $Z = Z^D + Z^C$ with $Z^D = \text{block diag}\{Z_{11}, Z_{22}, \ldots, Z_{nn}\}$.

2.5 Theorem (QBDD): An mXm matrix $Z=[Z_{ij}]$ partitioned as above into mXm blocks is said to be quasi-block diagonal dominant (QBDD) if any of the following conditions hold:

(i) there exists a real m-vector x>0 such that $[I_n - \hat{B}]\, x > 0$

(ii) the test matrix $[I_n - \hat{B}]$ is an M-matrix

(iii) if $\hat{B}$ has a dominant Perron eigenvalue then $\lambda_{PF}<1$.

Note also that any such Z is nonsingular on D.

We remark that a partitioned matrix Z is QBDD if it is GBDD (as long as the same matrix norms are used in forming the test matrices B, C, and $\hat{B}$.) This follows from the fact that for any matrix norm $||AB|| \le ||A||\,||B||$. Thus in general QBDD is a less conservative measure of weak coupling for transfer functions than either BDD or GBDD.

It should be noted that QBDD as discussed in Okuguchi [20] is a slightly more general notion of BDD than considered in Feingold and Varga [9] also because of consideration for applying any valid matrix norm for the individual subblocks. By comparison in [9] matrix norms for individual blocks are subordinate to vector norms on corresponding subspaces; thus QBDD allows for example the use of trace class norms such as the Frobenius norm to be employed.

3.0 Decentralized Control

Stability tests of the form of theorem 2.1 together with BDD (GBDD or QBDD) provide qualitative information about candidate designs. They do not, however, provide sufficient information in most cases to determine either: (i) whether a candidate stabilizing decentralized control exists or, (ii) given a nominal stabilizing decentralized control what design freedom is left to allow satisfying additional design criteria. In short these stability results alone do not provide sufficient quantitative information to suggest a unified method for design of decentralized control. The natural viewpoint in decentralized design based on weak coupling assumptions is that the design process itself becomes decentralized. That is the individual compensator terms $F_i(s)$ in $F(s)=\text{block diag}\{F_i\}$ can each be chosen individually subject to a nominal decoupled system model. The accuracy of this approximation must be known rather precisely for such methods to be effective. Indeed, in the case of general partitions of G(s), the goal of decentralizing the design process suggests the potential for different design techniques appropriate to the local control objectives to be used. An appropriate measure of weak coupling such as BDD must quantify the accuracy of the decoupled model approximation in a manner consistent with the partition employed. Estimates for the accuracy of decoupled transfer function models are discussed in section 3.2.

3.1 Stability

The application of diagonal dominance for transfer functions as first suggested by Rosenbrock was to provide sufficient conditions for condition (iii) of theorem 2.1 to hold thus permitting decentralization of the design process. The various extensions to this method primarily sought to extend the application of this method to a larger class of systems by application of the refined measures discussed in the previous section and by including the possibility of more general partitions of the transfer function matrices involved. These tests can be applied in several ways which we summarize in a theorem.

3.1 Theorem: With G(s) and H(s) each mXm as defined in (2.1)-(2.2) (resp. G and H are partitioned conformally) then if either of the following conditions hold:

(i) H and G are both (resp. block) diagonally dominant

(ii) H^{-1} and G^{-1} are both (resp. block) diagonally dominant

(iii) for F both asymtotically stable and minimum phase $F^{-1} + G$ is (resp. block) diagonally dominant

(iv) F both asymtotically stable and minimum phase $F + G^{-1}$ is (resp. block) diagonally dominant

(v) $R = I + FG$ is (resp. block) diagonally dominant

on the closed contour D then condition (iii) of theorem 2.1 holds. From now on (unless otherwise specified) when we refer to block diagonal dominance (BDD) we will mean the collection of techniques discussed in section 2.

3.2 Accuracy of the Decoupled Model Approximation for Weakly Coupled Systems

The practical design of decoupled feedback control in INA methods is possible due to a classical result of Ostrowski [23] which provides a bound on the distance between the inverse of the diagonal elements of a complex-valued matrix which is diagonally dominant and the diagonal elements of the matrix inverse (which are, of course, usually different). Let $H(G,F_k)$ be the closed loop transfer function resulting from the decoupled feedback

$$F_k=\text{diag}\{f_1,\dots,f_{k-1},0,f_{k+1},\dots,f_m\}$$

which is the transfer function from input u_k to output y_k with the kth loop open . Using Ostrowski's result it is possible to show if condition (i) of theorem 3.1 holds that

$$|h_{kk}(G,F_k) - g_{kk}| < \phi_k d_k < d_k, \tag{3.1}$$

for $k = 1,\dots,m$ where

$$d_k = \sum_{j\neq k}^{m} |g_{kj}(s)|, \tag{3.2}$$

$$\phi_k = \max_{j\neq k} \frac{d_j}{|f_j^{-1} + g_{jj}|}. \tag{3.3}$$

Inequality (3.1) provides an example of how the accuracy of the nominal decoupled model can be quantified. Moreover, the factors ϕ_k suggest how the accuracy of some local models can be effected by the choice of the other controllers. This suggests, for example, that local designs with more restrictive control objectives (i.e. requiring more accuarcy in the approximate decoupled model) should be considered last in the sequence of designs as long as $\phi_k<<1$ can be maintained over a significant frequency band. Alternately, as suggested by Rosenbrock, the use of the inverse transfer function model for design leads to an expression for the factors ϕ_k of the form (3.3) (in the case that (ii) of theorem 3.1 holds) but for which the denominator term is proportional to the loop gains f_j for moderately large gains. (This is the primary reason for Rosenbrock's preference for the inverse transfer function formulation.) The resulting bounds can be seen to strictly <u>improve</u> as the loop gains are chosen. In INA design the improved bounds can be applied directly to the relevant Nyquist locus for each individual loop to estimate the local stability margins.

Improved estimates for these bounds using GDD of theorem 2.4 were first suggested by Araki and Nwokah [2]. Later Nwokah [15] provided the construction of optimally sharp bounds as follows.

3.2 Theorem: With B and C defined as in (2.5), (2.6) assume C is <u>irreducible</u>. Then let λ_{PF} be the Perron root of $[I_m + CB^{-1}]$. Then the matrix $A - B + C$ will be a "locally minimal" semi M-matrix where A is a diagonal matrix formed as $A = \lambda_{PF} B$.

Remark: Recall that a semi M-matrix is a generalization of M-matrices with principal minors nonnegative. A semi M-matrix is said to be "locally minimal" [15] if $M - D$ is not even a semi M-matrix for any diagonal $D>0$.

Application of the above result provides a bound of the form (3.1) (by substitution of $B=\text{diag}\{b_i\}$ with $b_i = |f_i^{-1} + g_{ii}|$ and C as in (2.6))

$$|h_{kk}(G,F_k) - g_{kk}| < a_k |g_{kk}| \tag{3.4}$$

with $A = \text{diag}\{a_i\}$ as computed as in theorem 3.2. Clearly the inequality $a_k|g_{kk}|\leq d_k$ is guaranteed by GDD in theorem 2.4. This bound is however independent of the choice of the individual loop gains and provides therefore no guidance as to the relative importance of their selection.

For the case of more general partitioning of the transfer functions, bounds of this type have been provided by several authors. The present authors [3] derived a bound on the matrix infimum $(||A^{-1}||^{-1}=\inf_{x\neq 0}||Ax||/||x||)$ as

$$||[H_{kk}(G,F_k) - G_{kk}]^{-1}||^{-1} < \theta_k \phi_k ||G_{kk}^{-1}||^{-1}$$

$$< \theta_k ||G_{kk}^{-1}||^{-1} \quad (3.5)$$

where

$$\theta_k = \sum_{j\neq k} ||G_{kj}||/||G_{kk}^{-1}||^{-1}$$

$$\phi_k = \max_{j\neq k} \theta_j.$$

Limebeer provided a sharper bound for this quantity by using GBDD. More recently Ohta et al gave a bound of the form

$$||H_{kk}(G,F_k) - G_{kk}|| < \gamma_k ||G_{kk}|| \quad (3.6)$$

using the QBDD results and by appropriate generalization of the method of Araki and Nwokah [2]. They gave considerations for obtaining the factors γ_k in a manner which provide bounds irrespective of the choice of the compensators F_k.

It is worth emphasizing the utility of the bounds (3.5) and (3.6) in practical application of decentralized design. First, the bounds suggest that decentralization of the design process can be successful if the design methods used for each local design are robust with respect to the accuracy of the decoupled approximation. The modern application of multivariable design provides various ways to evaluate the robustness of control design with respect to uncertainty of the model input-output response. Thus for example, bounds of the form (3.5) and (3.6) can be translated into specifications on the minimum singular values of the matrix return difference associated with the individual decoupled designs [8]. Furthermore, resulting performance of the decentralized control can be readily estimated with an accuracy which represents the effect of the individual controllers on the weak coupling measure by the refined bounds given in (3.1) and (3.5). It is worthwhile noting that refined bounds for the metric in (3.6) but representing the effect of the other controllers have apparently not appeared in the literature.

It is also worth remarking that for general partitions and the resulting sequential (or decentralized) set of multivariable designs the choice of matrix norms in the definition of BDD and the bounds as discussed here is a non-trivial extension of the cases considered orginally by Rosenbrock [23] and later by Araki and Nwokah [2]. For instance the matrix infimum $||A^{-1}||^{-1}$ (used in BDD and GBDD) makes sense only (for A a square matrix) as an induced matrix norm subordinate to some vector norm $||.||$ on the appropriate subspace. Computation of $||A^{-1}||^{-1}$ for general l_p vector norms may be numerically illconditioned (depending on the conditioning of A with respect to inversion). As a result practical application of many of these results may be limited to euclidean vector norms on all subspaces. The resulting singular value analysis of the matrix blocks can be performed by well known algorithms with robust numerical properties. However in the definition of QBDD no such restriction on matrix norms appears necessary. Thus permitting more general interpretation of decoupled model accuracy and weak coupling via (3.6).

3.3 Design Methods and Discussion

The strength of the INA design method for multivariable design comes from its graphical interpretation which can be adequately supported by computer generated graphics. For the trivial matrix partitioning the Gerschgorin disks are inclusion regions for the matrix spectrum. Thus it is natural to suggest extending the method of characteristic loci design [21] to decentralized control using general Gerschgorin sets for partitioned matrices to generate inclusion regions for the system eigenloci. This approach was suggested by Nwokah [17] using GBDD and employing the structure of M-matrices. Subsequently, Limebeer suggested a similar generalization but focusing on graphical representation of these inclusion regions for design. In this section we will discuss alternatives for design methods for arbitrary partitions and highlight some limitations of the obvious choices.

In our opinion the successful use of block Gerschgorin estimates for the spectrum of a matrix in decentralized design based on the characteristic loci methods will be considerably limited in practice. Although application of the results of BDD to provide inclusion regions for the eigenloci of the composite model in terms of the eigenloci of the diagonal blocks is perhaps the most natural generalization of INA methods there are several deficiencies in this generalization;

(i) Generally the inclusion regions for partitioned matrices block Gerschgroin sets for a complex matrix $Z=[Z_{ij}]$; viz., for each $i=1,\dots,n$,

$$G_i = \{s \in C: ||(sI - Z_{ii})^{-1}||^{-1} < \sum_{j\neq i}^{n} ||Z_{ij}|| \}$$

$$G_i' = \{s \in C: ||(sI - Z_{ii})^{-1}||^{-1} < \sum_{j\neq i}^{n} ||Z_{ji}|| \}$$

(which of course coallesce in the case of GBDD or QBDD) are <u>not</u> disks except in the rather special case when the Z_{ii} are normal and the matrix norm employed is axis oriented. The more general shape of these sets may be difficult to determine.

(ii) The sets G_i and G_i' may be covered by disks for the purposes of graphical presentation as proposed by Limebeer [13]. However, such bounds (which depend on the eigenvectors of the blocks Z_{ii}) are not at all sharp and can be totally useless when there are near confluences in the eigenloci of the Z_{ii} as s varies over D.

(iii) As discussed in detail by Doyle and Stein [8] and by Postlethwaite et al [22] it is not possible to provide a useful notion of multivariable stability margin from characteristic loci plots without additional information (such as minimum and/or maximum singular values of certain transfer functions.) Thus the robustness question must be evaluated separately. The discussion of the last section being relevant.

Ohta et al [18] have recently extended a parametrization of a loop-at-a-time design procedure of Araki and Nwokah [2] to decentralized design. Their method exploits the natural ordering of strict positive and M-matrices together with a useful parametrization of the control structure. We will summarize these results in terms of the generalization of [18] for arbitrary transfer function partitioning using QBDD concepts (see [18] for a detailed comparison with the other BDD methods.) As stated the results reduce to the original method [2] for the case of the trivial partitioning of all transfer functions involved.

Application of the test for QBDD (as given in theorem 2.5) to the matrix Z=R, the matrix return difference of (2.3), may indicate stability via (v) of theorem 3.1. The test matrix $\hat{B} = (\hat{b}_{ij})$ of (2.11) is given in this case as,

$$\hat{b}_{ij} = \begin{cases} 0, & \text{for } i=j \\ \|R^C_{ij}(R^D_{jj})^{-1}\|, & \text{for } i\neq j \end{cases} \quad (3.7)$$

The theory of M-matrices suggests alternate test matrices $C=(c_{ij})$,

$$c_{ij} = \begin{cases} 0, & \text{for } i=j \\ \|G_{ij}\, G^{-1}_{jj}\|, & \text{for } i\neq j \end{cases} \quad (3.8)$$

and $D=\text{diag}\{d_1,\ldots,d_n\}$ with $d_i=\|R^C_{ij}(R^D_{jj})^{-1}\|$ be formed so that $\hat{B}\leq CD$ [18, eqn (3.20)] (elementwise). As is well known [10] M-matrices can be partially ordered which can be exploited in this case in the following way. Given a real nXn diagonal matrix $\Gamma=\text{diag}\{\gamma_i\}$ with $\gamma_i>0$ for i=1,...,n it can be seen if $\Gamma - C$ is a semi M-matrix then for any diagonal matrix $D<\Gamma^{-1}$ that $D^{-1} - C$ and $I - CD$ are both M-matrices. Thus stability can be guaranteed (via theorem 3.1) for all decentralized controllers which satisfy the individual bounds for k=1,...,n

$$d_k < 1/\gamma_k$$

where $d_k = \|G_{kk}F_{kk}[I_{m_k} + G_{kk}F_{kk}]^{-1}\|$.

To provide maximal design freedom here a choice of $\gamma_k= \lambda^{-1}_{PF}\{C\}$, the Perron root of C, is suggested for C an irreducible nonnegative matrix [12,17,18]. These bounds provide sufficient conditions for the return difference to be QBDD. We remark that if C is reducible in the sense of the Perron-Frobenius theory of nonnegative matrices then a reordering strategy is available which provides the appropriate estimates after a decomposition into irreducible submatrices [18].

A useful parametrization of the controller structure for decentralized design which exploits this property is suggested by Ohta et al. Let the diagonal block F_{kk} be $r_k X r_k$

$$F_{kk} = K_k P_k$$

for k=1,...,n with

$$P_k = \text{diag}\{p^k_1, p^k_2, \ldots, p_{r_k}^k\}.$$

Ohta et al [18] consider the idea of applying charcteristic loci design in a decentralized fashion and standard INA method for the choice of the individual loop gains p^k_i sequentially for each kth decentralized controller. Their formulas suggest the limitations we discussed earlier of decentralizing the characteristic loci method via block Gerschgorin estimates in terms of the required eigendecomposition of the diagonal blocks $G_{kk}K_k$; viz., the condition number of this eigendecomposition is readiliy apparent in the estimates obtained.

One final note on the application of INA methods for the local feedback designs F_k. The choice of the factor K_k suggested by Ohta may involve products of elementary types of controllers including PI and transformations, however considerations for choosing the K_k to enhance diagonal dominance of the individual terms K_kG_{kk} remains incomplete. In [3] a design of this type was considered in order to demonstrate that BDD used in conjunction with diagonal dominance estimates for the diagonal partitions can lead to enhanced estimates of stability.

3.4 Information-Sharing Control Structures and Decentralized Design

Techniques for decentralized design based on weak coupling of partitioned input-output models provide naturally the decentralized information patterns for control consistent with the partition chosen. However, as suggested by Ohta et al, the notion of overlapping decompositions (which was orginally popularized by Siljak for construction of vector Lyapunov functions) can be applied to the system transfer function matrix. The resulting design methods can provide a decoupled approach to design for a control system in which controllers are allowed to share information according to some restricted information pattern.

Previous application of overlapping decompostion has been focused on tranformations which expand and contract the state space for a given system model in order to provide constructive methods for Lyapunov stability tests for decentralized control. In the context of the present notion of weak coupling of the system input-output response Ohta et al suggested the possiblity for expansion and contraction of the space of system inputs and outputs [19]. We briefly summarize this circle of ideas in the following.

Given the feedback equations of (2.1) with G(s) (F(s)) a pXm (resp. mXp) transfer function we consider an expanded version of this system in terms of $\tilde{p}X\tilde{m}$ (resp. $\tilde{m}X\tilde{p}$) transfer function $\tilde{G}(s)$ (resp. $\tilde{F}(s)$) where $\tilde{m}>m$, $\tilde{p}>p$. In order to preserve stability properties of the original system in the expanded input-output space Ohta has restricted the expanding transformations to be of the form:

(i) $\tilde{G}\, U = V\, G, \qquad \tilde{F}\, V = U\, F$

(ii) or $V^+\, \tilde{G} = G\, U^+, \qquad U^+\tilde{F} = F\, V^+$

where V^+ (resp. U^+) is the left inverse of the $\tilde{p}Xp$ (resp. $\tilde{m}Xm$) rational matrix V (resp. U). Equivalently such a system can be represented in the "contracted" or original system input-output space as

$$G = V^+\, \tilde{G}\, U + M \quad (3.9)$$
$$F = U^+\, \tilde{F}\, V + K$$

where the rational matrices M and K must satisfy either:

(i) $MU^+ = 0,\ K\, V^+ = 0$

(ii) or $VM = 0,\ UK = 0.$

That QBDD transfer functions can be generated in the expanded input-output space is demonstrated by example in Ohta et al [19]. It is easily demonstrated how more general restrictions on the information pattern for control can be included as additional constraints in the choice of the expanding transformations U and V. An interesting example is included which highlights the possibility for enhanced decoupling of the input-output response of the closed loop system with decentralized control by employing an overlapping control structure designed by the above method; viz., 1) expand the input-output model consistent with the overlapping control structure, 2) perform decentralized design using QBDD methods in the expanded space, and 3) construct the overlapping control by contraction of the resulting decentralized control law to the original space of m inputs and p outputs.

Obviously the increased flexibility of these methods offer possibilities for improved performance and application in an expanded array of decentralized design problems. However caution should be exercised in the choice of transformations U and V satisfying the above equations so that illconditioning of the resulting estimates does not result. It is apparent that the successful application of these methods depends upon adequate consideration for resulting robustness and sensitivity properties [8].

3.5 Considerations for Approximation of Weakly Coupled Systems

It is clear from the development of Rosenbrock's work and the various extensions considered here for decentralized design that the central theme in providing a unified design methodology based on weakly coupled models is approximation of frequency response data (or equivalently the transfer functions involved.) Appropriate considerations for stability and robustness properties of feedback systems based on approximate transfer function models are made available via the Nyquist stability theory and the principle of the argument from complex analysis. We support the view in [4] based on the geometric ideas in [7] that a particularly appropriate form of Nyquist stability theory for such approximation questions involves natural topological considerations for transfer functions.

An alternate form of weak coupling which provides sufficient conditions for (iii) of theorem 2.1 was introduced in [4]. The idea here is to consider directly the question of approximation of transfer functions using a gap metric. This approach has also the potential advantage of including certain relative phase information between subsystem transfer function blocks which is lost through the use of the various matrix norms introduced in diagonal dominance tests. Moreover, this metric includes immediate considerations for robustness in terms of a new geometric stability margin for (possibly multivariable) transfer function models [6]. Since this new stability margin is discussed in detail in this proceedings [6] we will only highlight the significance for providing weak coupling estimates here.

Natural topological considerations for transfer functions with regard to stability can be provided from an abstract Nyquist locus for possibly multivariable plants introduced by Brockett and Byrnes [7]. The Nyquist locus considered here is a curve in a complex Grassmanian arising from consideration of the feedback equations (2.1) which we can write in the form

$$\begin{vmatrix} I_p & G \\ F & -I_m \end{vmatrix} \begin{vmatrix} y \\ e \end{vmatrix} = \begin{vmatrix} 0 \\ u \end{vmatrix} .$$

The abstract Nyquist contour is the image of the usual closed contour D under the map $\ker[I_p, G(s)]$ which can be thought of as a locus of m-dimensional subspaces of a p+m complex space. In this regard one can consider the subspace $\ker[F, -I_m]$ as representing the "critical point" for Nyquist stability testing. Metrics can be constructed from the notion of <u>principal angles</u> between subspaces which measure both the distance and nearness of intersection between pairs of subspaces [4-6]. In this setting "weak coupling" can be determined in terms of the distance between the abstract Nyquist curve for G(s) and one for $G^D(s)$, the decoupled approximate system. With the following notation we can state the weak coupling condition of [4-6]. Let $\underline{G}$ be the subspace $\ker[I_p, G]$ and $\underline{F} = \ker[F, -I_m]$. Let $\underline{\theta}(\underline{A},\underline{B})$ (resp. $\bar{\theta}(.,.)$) be the minimum (resp. maximum) principal angle between the pair of subspaces $\underline{A}$ and $\underline{B}$. Then [4-6] the condition

$$\bar{\theta}(\underline{G}^D,\underline{F}) \geq \underline{\theta}(\underline{G}^D, \underline{G}) \qquad (3.10)$$

for all s on D is sufficient for the "weak coupling" assumption (iii) of theorem 2.1 [5]. Inequality (3.10) provides the same quantitative information as for example (3.6) but in terms of the new angle metric.

We believe that this metric can offer considerable flexibility in various areas where approximation of transfer function models for feedback control synthesis is required. In particular their application to decentralization via weak coupling arguments is quite natural.

4.0 Conclusions

In this paper we have attempted to review the available theory and techniques for decentralized control system design based on transfer function models. The methods considered here employ a requirement for a weak coupling condition between subsystem models to hold. For the most part, available methods have been developed along the lines of the INA method of Rosenbrock. Various methods of testing for weak coupling in a frequency dependent setting have been reviewed. The utility of this assumption, should it be seen to hold, follows from several facts; (1) a decoupled model approximation can be used for design, (2) the resulting decentralized control employs an information pattern consistent with a natural weak coupling of the plant, and (3) the weak coupling measures permit a quantitative evaluation (in terms of transfer function models) of the resulting control system performance and robustness properties.

In contrast to weak coupling of state space models, design methods based on system frequency response have not gained wide acceptance due in part to a lack of understanding of the physical significance of the weak coupling notions employed. However, we suggest that considerations for the control of information flow between individual control computers and the system under control (i.e. decentralization) is naturally considered in terms of input-output models. We believe these methods are potentially useful for control because of several factors: (1) the uniqueness of input-output models (vs. state space models), (2) the significance of these measures for robust control synthesis.

Certain open questions remain in the application of these theoretical results to decentralized design. A major element in application of these methods for decentralized design is the choice of partitions for transfer functions. Theoretical considerations for optimal choice of disjoint partitions is discussed in Varga [27], but his considerations are for spectral estimates in terms of minimal Gerschgorin sets whereas for transfer function models other considerations may arise. Ohta et al [28] consider more general overlapping partitions by introducing "expanding and contracting" transformations on the space of inputs and outputs. It is also possible to consider recursive application of weak coupling tests for various partitions. In particular an example discussed in [3] indicated that a transfer function not diagonally dominant in the sense of the trivial parition was seen to be block diagonally dominant according to a certain partition for which the diagonal blocks where found to be diagonally dominant in the usual sense. As a result a completely decoupled stabilizing feedback was constructed for the transfer function based on a model which was "recursively" partitioned [3].

The remaining papers in this session serve to illustrate by example comparison of these and other methods for decentralized control.

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