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"Delight: LQG, a CAD System for Control System Design Using LQG Controller Structure"

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DELIGHT.LQG, a CAD System for Control Systems Design Using LQG Controller Structure[1]

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Abstract

A new CAD package for the design of control systems is presented. The package combines the powerful interactive optimization-based methodology of the DELIGHT system with the control theoretic design methodology known as LQG. The result is a powerful, self-contained design package which can be used to design controllers to practical engineering specifications in a highly interactive mode. An example from the design of a flight control system for a high performance aircraft is included.

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Summary

In this paper we describe a powerful optimization-based design methodology for CAD of multivariable control systems when the controller structure is determined following the principles of LQG theory. The methodology is currently implemented in a software CAD package named DELIGHT.LQG, which is the result of the marriage of the DELIGHT interactive optimization-based CAD system [4, 5] and certain parts of the ORACLS package for linear multivariable control [2], enhanced by additional algorithms for frequency response computation. With this design system, a control system designer can take advantage of recent powerful optimization algorithms from the DELIGHT system to automatically adjust design parameters of the LQG controller. These design parameters will usually be components of the various matrices used to describe the compensator, plant, or other system blocks. The designer may optimize arbitrary performance criteria as well as study the tradeoffs between multiple competing objectives, while simultaneously keeping several constraints specifications met.

For our purposes, control system design can be considered as a two-phase iterative process. The designer first utilizes a control theoretic methodology to define the structure of the controller and appropriate parametrization. Part of this first phase is the determination of appropriate models for the plant, servos, disturbances, etc. The second phase is primarily involved with determination of parameter values, so that certain engineering specifications are met. Several iterations may be necessary which should provide self-consistent designs with the structure selected during phase one.

It is significant to point out that in control systems design, system perfor-

mance and specifications are in general functions of the design parameters through the frequency and transient response description of the control system. For example, the design of a flight control system could have as *performance objective* the closed loop bandwidth of the system and as a *constraint specification* that the maximum plant output (at a specified output port) be less than some value. The design parameters could be an actuator gain and a rate feedback weight. Neither the performance objective, nor the constraint specification are explicit functions of the design parameters; this dependence is implicit through analyses of the control system equations. In particular, the closed-loop bandwidth can be evaluated by finding the 3 db point of the frequency response from a suitable analysis, while the maximum plant output can be determined from a transient analysis with prescribed input. Thus the performance and specification evaluations required to perform optimization of a feedback system often involve expensive system simulations. The DELIGHT.LQG system computes these system responses using simulation routines from ORACLS and from other sources.

Optimization techniques have been applied successfully to numerous design problems in various branches of engineering. However, quite often, the mathematical problem solved by the optimization algorithm may be remote from the real world problem the designer is facing. This is (obviously) due to the rigidity of the classical nonlinear programming problem which can be stated as

$$\min\{\ f(x)\ \mid\ g(x)\leq 0\ \} \tag{1}$$

where $f(x)$ is a cost or objective function to be minimized and $g(x)$ represents several inequality constraints and where x is the vector of design parameters.

This formulation fails to account for several important characteristics of a large class of design problems. First, typically in real life design applications, several (often conflicting) performance objectives are utilized. Second, constraint specifications are often relatively flexible and therefore moderate violation of a constraint should be acceptable. Third and more critical, the above formulation describes (in a quantitative way) only partially the knowledge a designer has about his problem. For example this formulation does not permit either a representation of intuitive knowledge about the design problem, or about the degree of confidence the designer has in the initial guess of design parameters, or about the significance of each specification as compared to others.

In this paper some of the ideas described in [7], and incorporated in current versions of DELIGHT at the University of Maryland, are employed to circumvent the above limitations. The basis for this methodology is the partition of the various design specifications in an engineering design problem into three categories: hard constraints, soft constraints and objectives.

To better illustrate this classification of engineering specifications, let us consider the multivariable control system loop depicted in fig. 1 below. $P(s)$ represents the given plant, while $C(x,s)$ is the controller to be designed and x describes a finite set of design parameters. In particular the structure of the controller has already been decided. For the purposes of the present paper the structure of $C(x,s)$ is the one implied by LQG theory [1]. The values of the design parameters x are to be chosen so that a number of engineering specifications are satisfied. Typically these specifications include stability, sufficient robustness, and properties of various time and frequency responses.

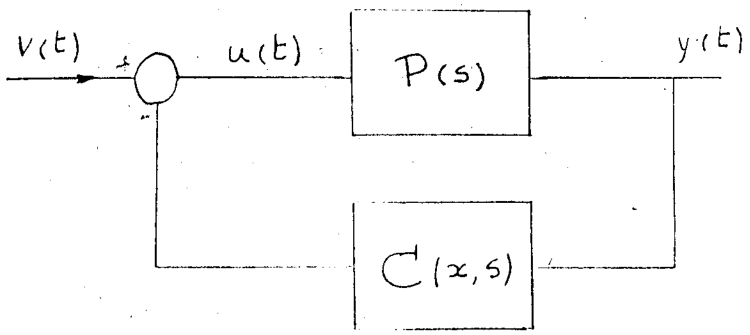

Figure 1. Standard multivariable control loop

Obviously requirements such as stability are imperative, whereas often specifications of the latter types are inherently imprecise, i.e. are not required to meet any precise numerical values. In the usual case where several specifications compete against each other, several "tradeoff" solutions may be equally of interest.

The first class is the class of *hard constraints*. An obvious example is the stability requirement. That is the design parameters must be such that all the closed loop poles be in the left half plane, i.e.,

$$\operatorname{Re} p_i(x) \leq 0 \quad \forall i \quad . \tag{2}$$

There is clearly no point attempting to improve, say, transient properties of the control system unless stability has been insured. Another type of hard constraint relates to physical realizability within the frame of the chosen structure. This may include constraints on the sign and the magnitude of some controller gains, such as

$$0 \leq x^j \leq x^j{}_{\max}. \tag{3}$$

Thus, a specification is considered a hard constraint if its satisfaction is considered essential and hence of the *utmost priority*.

The second class is the class of *soft constraints*. A good such example is the requirement of stability under plant uncertainties. It is well known that (see, e.g. [8]), if the nominal closed-loop system is stable and if

$$\underline{\sigma}(I+(P(j\omega)\ C(x,j\omega)^{-1}) \geq b(\omega) \quad \forall\omega \tag{4}$$

(where $\underline{\sigma}$ indicates the minimum singular value) then the closed-loop system will still be stable if the actual plant is not exactly $P(s)$ but rather

$$P(s) = (1+L(s))P(s) \tag{5}$$

provided that $L(s)$ satisfies

$$\overline{\sigma}(L(j\omega)) < b(\omega) \quad \forall\omega. \tag{6}$$

Since $L(s)$ is unknown, a small violation of this specification can be quite acceptable, although no violation is preferable. Another example of a soft constraint is a specification of a maximum allowed overshoot for some step response, say

$$s(x,t) \leq s_{\max} \quad \forall t. \tag{7}$$

Again, a slight violation of this constraint would probably not jeopardize the value of the design, even though a design satisfying the constraint would be preferable. On the other hand, achieving an overshoot much smaller than $s_{\max}$ may not improve the value of the design. In short, a soft constraint is a specification involving a desired or target value, that the design should try to approach and reach if possible, but such that no further gain would be obtained if the specification "overachieved" its target value.

The third class of specifications is the class of *objectives*. A possible objective for control system design is to minimize the closed loop sensitivity to disturbances and changes in the plant, which can be expressed as

$$\max \underline{\sigma}(I+P(j\omega)\ C(x,j\omega)) \quad \forall \omega \in \Omega \tag{8}$$

where Ω is the range of frequencies over which low sensitivity is sought. Another example of an objective (which could coexist with the above in a multiobjective formulation) is to minimize the integral of the square of the error of a step response. In short, an objective is a specification for which some quantity should be minimized or maximized.

Since soft constraints may be traded off by the designer, it is important to specify the relative importance of these constraints to the optimization algorithm of DELIGHT.LQG. A natural way of indicating the relative importance of constraints (and also performance objectives) is by having the designer specify two values for each: a *good* value and a *bad* value. The meaning of these values is limited to the following understanding: having all of the various objectives and soft constraints achieve their corresponding good values should provide the same level of "satisfaction" to the designer for each, while achieving the bad values should provide the same level of dissatisfaction. This provides a very simple way to do trade off analyses: if a designer is unhappy with the performance level achieved by a particular objective or constraint, he simply changes what he considers to be satisfactory or unsatisfactory by adjusting the good and bad values, via the *setgood* and *setbad* commands, and then resumes execution of the optimization.

The algorithm DELIGHT.LQG provides belongs to a family of methods for constrained optimization known as *Methods of Feasible Directions.* For further details on this algorithm we refer to [6].

The control system design methodology utilized in DELIGHT.LQG is the Linear Quadratic Regulator theory [1]. This provides a certain controller structure with several well known properties (such as stability, robustness, etc.) embedded in the design (automatically). The novelty of our approach lies in the selection of the *design parameters* which permit the development of a consistent design with all the benefits of LQG. To illustrate this, let us consider first the design of a state feedback controller.

The system model is given by

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{9}$$

$$y(t)=Cx(t)+Du(t)$$

where A is $n\times n$, B is $n\times m$, C is $p\times n$ and D is $p\times m$. Thus $P(s)$ in fig. 1 is

$$P(s)=C(sI-A)^{-1}B+D. \tag{10}$$

The feedback controller used is (refer to fig. 1)

$$u(t)=-Kx(t)+v(t) \tag{11}$$

where the gain k is computed from LQ theory as follows. Since the algebraic Riccati equation

$$\Pi_\alpha(A+\alpha I)^T+(A+\alpha I)\Pi_\alpha-\Pi_\alpha BR^{-1}B^T\Pi_\alpha+Q=0 \tag{12}$$

$$K=R^{-1}B^T\Pi_\alpha$$

where $Q=Q^T\geq 0$, $R=R^T\geq 0$ are our *design parameters.* The algorithm continuously adjusts these parameters so as to satisfy certain specifications on the time or frequency response of the closed loop system depicted on fig. 1. These specifications include placing the step response between two given functions of time, satisfying other specifications on maximum output, bandwidth, etc. The choice of design parameters frees completely the designer from the usual burden of LQG design: "choose appropriate Q, R, so that desired closed loop performance results". The scalar α controls the stability of the closed loop. In this configuration, the controller $C(x,s)$ of fig. 1 becomes

$$C(x,s)=K(Q,R) \quad (13)$$

In the more general case of output feedback, the controller structure includes (12), (13), but the control values are

$$u(t)=-Kz(t)+v(t) \quad (14)$$

where $z(t)$ is described by

$$\dot{z}(t)=Az(t)+Bu(t)+K_o(y(t)-Cz(t)) \quad (15)$$

The gain K_o is computed as follows. First we solve the algebraic Riccati equation

$$(A+\alpha_o I)\Pi_o+\Pi_o(A+\alpha_o I)^T-\Pi_o C^T R_o^{-1} C\Pi_o+Q_o=0 \quad (16)$$

then

$$K_o=\Pi_o C^T R_o^{-1} . \quad (17)$$

Equations (15), (16), and (17) construct a state observer, following the method of Baras and Krishnaprasad [3], as limit of Kalman filter when both state and observation noise become zero.

In the paper, examples of the methodology as applied to high performance aircraft will be presented.

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