Proceedings of the 35th Chinese Control Conference
July 27-29, 2016, Chengdu, China

Attitude synchronization of multiple rigid bodies in SE(3) over proximity networks

Juan Deng[1], Zhixin Liu[1], Lin Wang[2], John S. Baras[3]

1. Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, CAS, Beijing 100190, P. R. China
E-mail: djbolatu@163.com, lzx@amss.ac.cn

2. Department of Automation, Shanghai Jiao Tong University, and the Key Laboratory of System Control and Information Processing, Ministry of Education, Shanghai 200240, China
E-mail: wanglin@sjtu.edu.cn

3. Department of Electrical and Computer Engineering and the Institute for Systems Research, University of Maryland, College Park, MD 20742, USA.
E-mail: baras@umd.edu

Abstract: This paper investigates the attitude synchronization of a group of moving rigid bodies whose states including positions and rotation matrices evolve in the Lie group $SE(3)$. The commonly used circular neighborhood is adopted, and the neighbor graphs are described by proximity networks. In order to avoid introducing discussions on the abrupt changes of neighbor graphs, a dwell time is introduced. The distributed sampled-data control law for the angular velocity is designed. Under the assumptions on the initial states, we provide sufficient conditions for the attitude synchronization of multiple rigid body system. Compared with the existing results, our condition depends only on the neighborhood radius, the moving velocity and the dwell time.

Key Words: rigid body, SE(3), attitude synchronization, sampled-data control, proximity network

1 Introduction

An important topic in the investigation of multi-agent systems is how to design the distributed control for each agent such that all agents coordinate to finish a complicated task or reach certain desired behaviors (cf. [1–4]). In recent studies on coordination control, most results are obtained in Euclidean space. However, many practical systems such as underwater vehicles, satellites and UAVs evolve in non-Euclidean spaces, which requires us to pay more attention on the systems where the agents evolve in non-Euclidean manifolds.

Compared with the mass model in which the objects are considered as a point with mass but without shape or size, the rigid body model takes the shape and size into account. Thus, the motion of rigid bodies includes both the rotation and the translation. The rotation matrix can globally and uniquely represent the rotation of rigid bodies, and all rotation matrices form the Lie group $SO(3)$. Some results have been given on the attitude synchronization (i.e., the rotation matrices converge to the same sate) of rigid bodies in $SO(3)$ (cf. [5–10]). For moving rigid bodies where both the rotation and the translation are taken into consideration, the pair of positions and rotation matrices of rigid bodies form $SE(3)$. $SE(3)$ is also a Lie group. Some results have been given for the pose synchronization (i.e., the positions and the rotation matrices, respectively, reach the same state) of $SE(3)$. For example, in [11], the authors considered the pose synchronization problem of rigid bodies in $SE(3)$ where the networks are undirected. In [12, 13], the authors studied the pose synchronization problem of rigid bodies in SE(3) based on the passivity-like property of the kinematics over directed networks. In [14], we addressed the pose synchronization problem with switching bidirectional networks. So far, almost all results on the attitude or pose synchronization of rigid bodies require that the communication graphs satisfy a certain connectivity condition. Although network connectivity has been widely studied in the area of wireless and ad hoc networks[15, 16], these results can not be employed for the synchronization analysis where the topology of networks may change with the agents' states. How to verify or guarantee such conditions is a challenging issue.

In this paper, we consider the attitude synchronization of the moving rigid bodies where the circular neighborhood is adopted. The pair of rigid bodies is called neighbors if and only if their Euclidean distance is less than a pre-specified radius. It is clear that the motion of rigid bodies may result in abrupt changes of neighbor graphs. In order to avoid in-

This work was supported by the National Natural Science Foundation of China under grants 61273221, and the National Key Basic Research Program of China (973 program) under grant no. 2014CB845302.

troducing discussions on neighbor graphs, a dwell time is introduced. As a consequence, each rigid body can only receive the sampled-data information from neighbors. The sampled-data technique has wide applications in the fields of communications and wireless networks. It is worth mentioning that the potential function approach which is commonly used to guarantee the connectivity of neighbor graphs (cf. [17–19]) are not suitable for the sampled-data case because the connectivity of the networks might be lost between sampling instants. In this paper, we consider a simpler case where the translational velocity keeps a constant. We design the distributed control law for the angular velocity based on the nearest neighbor rules. By relying on the rotation vector representation for rotation matrices and the estimation of some characteristics on the initial states, we present a sufficient condition for attitude synchronization under restrictions on the initial states. Our conditions only depend on the neighborhood radius and the moving speed and the dwell time without any assumption on the neighbor graphs.

The rest of this paper is organized as follows. In Section 2, we provide the problem statement. In Section 3, we present the main results of this paper. We make concluding remarks in Section 4.

2 Problem Statement

2.1 The Rigid Body

The motion of the rigid body includes translation and rotation. For a given rigid body $\mathfrak{R}$, we denote the world frame and the body-fixed frame as Σ_w and Σ', respectively. For simplicity of expressions, we assume that Σ_w and Σ' are right-handed and Cartesian. The position of the rigid body $\mathfrak{R}$'s center in the world frame Σ_w is denoted as $p \in \mathbb{R}^3$. The attitude of $\mathfrak{R}$ is represented by the rotation of the frame Σ' relative to Σ_w, and is denoted as the rotation matrix $R \in \mathbb{R}^{3\times 3}$. It is clear that the rotation matrix satisfies the equations $RR^T = I$ and $det(R) = 1$ with $det(\cdot)$ being the determinant of the corresponding matrix. Denote the set of all rotation matrices as $SO(3)$, i.e., $SO(3) = \{R \in \mathbb{R}^{3\times 3} : RR^T = I, det(R) = 1\}$. $SO(3)$ is a Lie group. The set of pairs of positions and rotation matrices is denoted as $SE(3)$, which is the product space of $\mathbb{R}^3$ with $SO(3)$, i.e., $SE(3) = \{(p, R) : p \in \mathbb{R}^3, R \in SO(3)\}$. The set $SE(3)$ defines the configuration of moving rigid bodies, and it is also a Lie group.

Since the rotation matrix is determined by nine variables with six constraints, it is generally complicated to investigate the attitude problem of rigid bodies using rotation matrices even though it can globally and uniquely represent the attitude of the rigid body. In order to solve this issue, the parameterized representations of rotation matrices are commonly adopted. In this paper, we study the attitude coordination of multiple rigid bodies by using the rotation vector $x \in \mathbb{R}^3$ rather than the rotation matrix to represent the attitude of rigid bodies. The rotation vector x evolves in a 3-dimensional Euclidean space, and it can be equivalently written as $x = \theta k$, where $k \in \mathcal{S}^2 = \{k \in \mathbb{R}^3 : k^T k = 1\}$ is called the rotational axis, and θ denotes the rotation angle around the axis k. If the angle θ is restricted in the interval $[0, \pi)$, then the rotation vector can uniquely represent the rotation matrix. In this paper we assume $\theta \in [0, \pi)$. The rotation vector can be obtained by the logarithm calculation of the matrix R, i.e.,

$$\widehat{x} = \log(R), \tag{1}$$

where the notation $\widehat{x}$ denotes the skew symmetric matrix generated by the vector $x = [x_1, x_2, x_3]' \in \mathbb{R}^3$, i.e.,

$$\widehat{x} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \tag{2}$$

2.2 System with Multiple Rigid Bodies

This paper considers a multi-agent system composed of n rigid bodies (or agents) labeled $1, 2, \cdots, n$, all moving in a three dimensional Euclidean space. The frame fixed on the rigid body i $(i = 1, 2, \cdots, n)$ is denoted as Σ_i. The position of the rigid body i in the world frame at time t $(t \geq 0)$ is denoted as $p_i(t)$, and the rotation matrix is denoted as $R_i(t)$ which evolves in the $SO(3)$. The velocity of the rigid body i is denoted as $V_i(t) = (\upsilon_i(t), \omega_i(t))$, where $\upsilon_i(t) \in \mathbb{R}^3$ and $\omega_i(t) \in \mathbb{R}^3$, respectively, denote the linear and angular velocities of the rigid body i in the world frame Σ_w at time t. The velocity $V_i(t)$ can be equivalently expressed as the following matrix

$$\widetilde{V}_i(t) = \begin{bmatrix} \widehat{\omega}_i(t) & \upsilon_i(t) \\ 0 & 0 \end{bmatrix},$$

where the notation $\widehat{\cdot}$ is defined in (2).

Denote an alternative expression for $(p_i(t), R_i(t))$ is the following 4×4 matrix

$$g_i(t) = \begin{bmatrix} R_i(t) & p_i(t) \\ 0 & 1 \end{bmatrix}.$$

The motion of the rigid body i $(i = 1, 2 \cdots, n)$ can be represented by the following kinematic model :

$$\dot{g}_i(t) = g_i(t)\widetilde{V}_i(t),$$

or, equivalently

$$\begin{cases} \dot{R}_i = R_i\hat{\omega}_i \\ \dot{p}_i = R_i v_i \end{cases}. \tag{3}$$

where the translation velocity v_i and the angular velocity ω_i are the control inputs. More details on the motion of the rigid bodies in *SE(3)* can be found in [20].

The purpose of this paper is to design the distributed control law for v_i and ω_i based on the local information such that the system reaches the attitude synchronization. Here by attitude synchronization we mean that for any two agents i and j, their rotation matrices satisfy the following equation,

$$R_i(t) - R_j(t) \to O_{3\times 3}, \quad as \quad t \to \infty.$$

2.3 Proximity Networks

The information transfer between agents plays an important role for the evolution of the system. In this paper, we adopt the commonly used circular neighborhood. The pair of two agents i and j are called neighbors if and only if their relative distance p_{ij} is less than a pre-defined radius r_n. In general, $p_{ij} = R_i^{-1}(p_j - p_i)$. In this paper, we assume that each rotation axis k_i pass through the origin of the body-fixed frame Σ_i for simplicity of analysis. For such a case, by [20] it can be seen that $p_{ij} = p_j - p_i$. The neighbor relations may vary with time when the agents move in three dimensional space. We use a graph sequence $G_t = (V, E_t)$ to describe the time-varying neighbor relations, where $V = \{1, 2, \cdots, n\}$ is the set of vertices, and $E_t = \{(i,j) : \|p_{ij}\| < r_n\}$ denotes the edge set. The graph G_t is undirected. The neighbor graphs formed in such a manner are distance-induced, and also called geometric graphs or proximity networks. We use $\mathcal{N}_i(t)$ to denote the set of the agent i's neighbors at time t, i.e., $\mathcal{N}_i(t) = \{j : (i,j) \in E_t\}$. Denote the cardinality of the set $\mathcal{N}_i(t)$, i.e., the degree of the agent i, as $d_i(t)$. It is clear that $d_i(t) \geq 1$ since each agent is a neighbor of itself.

The algebraic properties of the graph G_t are used for our analysis. For the graph G_t, the maximum and minimum degrees are defined as $d_{max}(t) = \max_{1\leq i\leq n} d_i(t)$ and $d_{min}(t) = \min_{1\leq i\leq n} d_i(t)$. The degree matrix is defined as $D(t) = diag(d_1(t), d_2(t), \cdots, d_n(t))$, and the Laplacian $L(t) = [l_{ij}(t)]$ is defined as

$$l_{ij}(t) = \begin{cases} d_i(t) - 1, & \text{if } i = j; \\ -1, & \text{if } i \neq j \text{ and } j \in \mathcal{N}_i(t) \\ 0, & \text{otherwise.} \end{cases}.$$

The normalized Laplacian is defined as $\mathcal{L} = D^{-1/2}LD^{-1/2}$. It is clear that $\mathcal{L}$ is nonnegative definite and 0 is one of the eigenvalues. We denote the eigenvalues of $\mathcal{L}$ according to a nondecreasing order as $0 = \lambda_0(t) \leq \lambda_1(t) \leq \cdots \leq \lambda_{n-1}(t)$. The spectral gap of G_t is defined as $\overline{\lambda}(t) = \max\{|1 - \lambda_1(t)|, |1 - \lambda_{n-1}(t)|\}$.

2.4 Distributed Control Design

What we concern is the coordination of the attitude of multiple rigid bodies whose closed-loop dynamics is affected by the control law of the angular velocity. In order to state our results clearly, we consider a simpler case where we just need to design the distributed control law of the angular velocity for each agent, while the linear velocity of the rigid bodies keep constant.

By the definition of neighbor relations, we see that the neighbor graphs may change abruptly when the agents move in three dimensional space. In order to avoid introducing discussion for the neighbor relations, we introduce the dwell time by assuming that the neighbor relations are only updated at sampled-data instants denoted as $t_0(= 0), t_1, t_2, \cdots$ with the dwell time τ_n, and keep unchanged in the sampled-data periods. The sampled-data instants satisfy $t_{k+1} - t_k = \tau_n$ for $k = 0, 1, \cdots, n$. At discrete-time instant t_k, each agent can receive the information of rotation matrices of its neighbors. For the rigid body i, it can receive the following sampled-data information at time t_k

$$\{R_j(t_k) : j \in \mathcal{N}_i(t_k), t_k \geq 0\}. \tag{4}$$

It is clear that the data receiving, storage and processing are generally fulfilled by computers, and the introduction of the sampled-data information makes it more feasible for the rigid bodies to use and process the data.

Remark 1 *In this paper, we assume that each agent can obtain the absolute information of the attitude of its neighbors, which can actually realized by either of the two following manners [21]. 1) For the agent i, its neighbor $j \in \mathcal{N}_i(t)$ can directly transmit its measurement $R_j(t)$ to the agent i. 2) We assume that the agent i can receive the relative information of the rotation matrix R_{ji} from its neighbor $j \in \mathcal{N}_i$. The agent i can calculate the rotation matrix R_j of its neighbor j by using the relation $R_j = R_iR_{ji}^T$.*

Remark 2 *By the uniqueness of the rotation vector representation for the rotation matrix when $\theta \in [0, \pi)$, we can assume that the agent i can receive the rotation vector of its neighbors at time t_k, i.e.,*

$$\{x_j(t_k) : j \in \mathcal{N}_i(t_k), t_k \geq 0\}. \tag{5}$$

The representation of rotation vectors brings convenience for our theoretical analysis.

For the rigid body i, we adopt the following distributed control law based on the sampled-data information (5) for $t \in [t_k, t_{k+1})(k = 0, 1, \cdots)$,

$$\omega_i(t) = \frac{1}{\tau_n d_i(t_k)} \sum_{j \in \mathcal{N}_i(t_k)} (x_j(t_k) - x_i(t_k)), \tag{6}$$

and the translational speed $v_i(t)$ is taken as a fixed vector $v_i(t) = v_0 \in \mathbb{R}^3$. Substituting (6) into (3), we have

$$R_i(t) = R_i(t_k) \exp \left\{ \frac{t - t_k}{\tau_n d_i(t_k)} \sum_{j \in \mathcal{N}_i(t_k)} (\hat{x}_j(t_k) - \hat{x}_i(t_k)) \right\},$$

that is

$$e^{\hat{x}_i(t)} = e^{\hat{x}_i(t_k)} \exp \left\{ \frac{t - t_k}{\tau_n d_i(t_k)} \sum_{j \in \mathcal{N}_i(t_k)} (\hat{x}_j(t_k) - \hat{x}_i(t_k)) \right\}. \tag{7}$$

We know that for any two matrices A and B, $e^A e^B = e^{A+B}$ when the two matrices are commutative. Therefore, in order to simplify the above dynamical system, we need $\hat{x}_i(t_k)$ and $\hat{x}_j(t_k)$ to be commutative, and the commutativity of $\hat{x}_i(t_k)$ and $\hat{x}_j(t_k)$ can be realized by imposing the following assumption on the initial rotation axis of all agents.

Assumption 1 *The rotation axis of all rigid bodies at the initial time instant are mutually parallel.*

Under Assumption 1, $x_i(0)$ is parallel to $x_j(0)$ for any i, j, which lead to $x_i(0) \times x_j(0) = 0$. From simple verification, we know $\hat{x}_i(0)$ and $\hat{x}_j(0)$ are commutative. By (7), we have $\hat{x}_i(t_k)\hat{x}_j(t_k) = \hat{x}_j(t_k)\hat{x}_i(t_k), \forall k$ and

$$\hat{x}_i(t) = \left(1 - \frac{t - t_k}{\tau_n}\right) \hat{x}_i(t_k) + \frac{t - t_k}{\tau_n d_i(t_k)} \sum_{j \in \mathcal{N}_i(t_k)} \hat{x}_j(t_k),$$

then

$$x_i(t) = \left(1 - \frac{t - t_k}{\tau_n}\right) x_i(t_k) + \frac{t - t_k}{\tau_n d_i(t_k)} \sum_{j \in \mathcal{N}_i(t_k)} x_j(t_k). \tag{8}$$

At sampled-data instant $t = t_{k+1}$, the rotation vector $x_i(t)$ evolves according to the following equation,

$$x_i(t_{k+1}) = \frac{1}{d_i(t_k)} \sum_{j \in \mathcal{N}_i(t_k)} x_j(t_k). \tag{9}$$

Denote $x(t) = [x_1^T(t), x_2^T(t), \cdots, x_n^T(t)]^T$. For $t \in [t_k, t_{k+1})$, the (8) can be rewritten into the following equation

$$x(t) = \left[(1 - \frac{t - t_k}{\tau_n}) \otimes I_3\right] x(t_k) + \left[\frac{t - t_k}{\tau_n} Q(t_k) \otimes I_3\right] x(t_k), \tag{10}$$

and for $t = t_{k+1}$, the equation (9) has the following equivalent form,

$$x(t_{k+1}) = (Q(t_k) \otimes I_3) x(t_k), \tag{11}$$

where the average matrix $Q(t_k) = (q_{ij}(t_k))$ is defined as

$$q_{ij}(t_k) = \begin{cases} \frac{1}{d_i(t_k)}, & \text{if } (i, j) \in E_{t_k} \\ 0, & \text{otherwise} \end{cases}. \tag{12}$$

By (8) and (9), we see that the neighbor relations plays a vital role for the evolution of the rotation vector of all agents, while the neighbor relations are determined by the relative position of the agents. By the constant control design for $v_i(t)$, we obtain the following equation for the position of the agents,

$$\dot{p}_i(t) = R_i(t) v_0. \tag{13}$$

3 Main Results

The purpose of this paper is to analyze the dynamical behavior of the closed-loop system (8), (9) and (13), and establish conditions for attitude synchronization of the rigid bodies.

In the analysis of the attitude synchronization, what we concern is the convergence of the maximum dissimilarity of the attitudes $\delta(R(t)) = \max_{i \neq j} \parallel R_i(t) - R_j(t) \parallel$. However, as we mentioned in Section 2, the representation of the rotation vector brings convenience for our analysis. We first show that $\delta(R(t))$ can be bounded by the maximum dissimilarity of the rotation vectors.

Lemma 1 *There exists a positive constant L such that $\delta(R(t))$ satisfies the following inequality,*

$$\delta(R(t)) \leq L \delta(x(t)), \ \forall \ t \geq 0,$$

where $\delta(x(t)) = \max_{1 \leq i,j \leq n} \parallel x_i(t) - x_j(t) \parallel$.

The proof of the above lemma is omitted due to space limitations.

By Lemma 1, we see that for any two agents, the difference of the rotation matrices is bounded by the difference of the rotation vectors which evolve according to equation (11). Under the assumption that the change of the average matrix $Q(t)$ in comparison with $Q(0)$ is not too much, we present the following result for the evolution of the rotation vectors.

Lemma 2 *[22] Let $\{G(t_k), k \geq 0\}$ be the sequence of neighbor graphs, with the corresponding normalized Laplacian $\mathcal{L}(t_k)$ and the average matrix $Q(t_k)$. If $\parallel Q(t_k) -$*

$Q(0) \parallel \leq \varepsilon$ *for some* $\varepsilon > 0$*, then for* $\{x(t_k)\}$ *recursively defined by* (11) *we have*

$$\delta(x(t_{k+1})) \leq \sqrt{2}\kappa(\bar{\lambda}(0) + \kappa\varepsilon)^k \parallel x(0) \parallel, \ k \geq 0$$

where $\bar{\lambda}(0)$ *is the spectral gap of* G_0*, and* $\kappa = \sqrt{\frac{d_{max}(0)}{d_{min}(0)}}$.

By Lemma 2, we see that in order to analyze the attitude synchronization of the rigid bodies, we need to estimate some characteristics including the initial degree, the spectral gap of the initial neighbor graph and $\|x(0)\|$, and deal with the difference of the average matrices $\|Q(t_k) - Q(0))\|$. The later will be dealt with in the proof of Theorem 1. For the estimation of the characteristics of the initial states, if we do not put any assumption on the initial distribution of the agents, then our analysis can only be proceeded from worst case, and the results will be very conservative. In order to overcome this issue, we introduce the following random assumptions on the initial states of all agents.

Assumption 2 *1) The positions of all rigid bodies at the initial time instant are mutually independent; 2) For all rigid bodies, the initial positions are uniformly and independently distributed(u.i.d.) in the unit cube* $[0, 1]^3$*; The initial rotation angles are u.i.d. in* $[0, \pi)$.

Under Assumption 2, we can obtain the estimation for the characteristics of the initial states, which are presented by the following lemma.

Lemma 3 *[24]Assume that the neighborhood radius satisfies the condition* $\sqrt[7]{\log^3 n/n} \ll r_n \ll 1$*. Then under Assumption 2, the following results hold almost surely for large* n*,*

1) The number of agents in the set $\mathcal{R}_i$ *is bounded by* $R_{max} = 8n\pi\eta_n r_n^3(1 + o(1))$.

2) The minimum and maximum degrees of the initial neighbor graph satisfy $d_{min}(0) = \frac{n\pi r_n^3}{6}(1 + o(1))$ *and* $d_{max}(0) = \frac{4n\pi r_n^3}{3}(1 + o(1))$.

3) The spectral gap of the initial neighbor graph satisfies $\bar{\lambda}(0) \leq 1 - \frac{r_n^2}{1568\sqrt{7}\pi}(1 + o(1))$.

Now, we are in a position to deal with the change of average matrices $\|Q(t_k) - Q(0)\|$. By (12), we see that the matrix $Q(t)$ is defined via the position of the agents at time t. Assume that the distance between any two agents i and j satisfies

$$\big|\|p_{ij}(t_k)\| - \|p_{ij}(0)\|\big| \leq \eta_n r_n, \tag{14}$$

where $\eta_n \in (0, 1)$ is a positive constant to be determined later. If at the initial time instant the distance between i and j satisfies $\|p_{ij}(0)\| < (1 - \eta_n)r_n$, then by (14) we have $\|p_{ij}(t_k)\| < r_n$; Otherwise, if $\|p_{ij}(0)\| > (1 - \eta_n)r_n$, then $\|p_{ij}(t_k)\| > r_n$. Hence, for the rigid body i, the change of its neighbors during evolution is bounded by the following set,

$$\mathcal{R}_i = \{j : (1 - \eta_n)r_n \leq \|p_{ij}(0)\| \leq (1 + \eta_n)r_n\}$$

Now, we present the conditions on the neighborhood radius and the moving velocity to guarantee that the equation (14) holds. It is clear that by (13), the position of the agents is affected by the rotation matrix, and the rotation matrix is determined by the rotation matrices of the corresponding neighbors; while the neighbors are defined via the distance between agents. We solve the coupled relationship by the following theorem.

Theorem 1 *If the neighborhood radius* r_n*, the fixed linear velocity* υ_0 *and the dwell time* τ_n *satisfy the following conditions:*

$$\sqrt[7]{\frac{\log^3 n}{n}} \ll r_n \ll 1, \quad \| \upsilon_0 \| \, \tau_n \leq \frac{1}{3136\sqrt{7}\pi} \cdot \frac{\eta_n r_n^3}{\log n}$$

where the parameter η_n *is taken as* $\eta_n = \frac{r_n^2}{864 \cdot 3136\sqrt{14}\pi}(1 + o(1))$*. Then under Assumption 1 and Assumption 2, the system* (3) *with the sampled-data control law* (6) *reaches attitude synchronization almost surely for large* n.

The proof of the above theorem is omitted due to space limitations.

4 Concluding Remarks

In this paper, we investigated the attitude synchronization problem of the moving rigid bodies in $SE(3)$, where the rigid bodies are connected via dynamical proximity networks. The dwell time was introduced when updating information from neighbors. We designed the distributed control algorithm for the angular velocity based on the local sampled-data information from neighbors, and presented sufficient conditions to guarantee attitude synchronization of rigid bodies without assuming connectivity of neighbor graphs. Many problems deserve to be further investigated, for example, how to design the control law for the linear velocities based on the relative position information from neighbors and how to establish conditions for pose synchronization of rigid bodies? If the agents can only receive the relative information from neighbors, then how do we design the distributed control law for agents to reach synchronization?

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