

# Model-Based Systems Engineering Applied to the Trajectory Planning for Autonomous Vehicles

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**Abstract**— Passing maneuver is a complex driving maneuver critical to success of autonomous vehicles. It becomes more challenging in two-way roads. In this study, a passing scenario with three vehicles is considered where car 1, an autonomous vehicle (AV), is moving behind car 2 (human-driven) in the same lane and car 3 (human-driven) is part of the oncoming traffic in the adjacent lane. The primary goal is to develop a framework to analyze measurement-based decision-making strategies for the AV satisfying driving safety constraints and while considering collaboration amongst drivers. The problem is structured using SysML to build a modular architecture with clearly defined interfaces and allocated behaviors. Mathematical models have been modeled in MATLAB. The integrated model of the car passing problem was then executed in MATLAB to study the effect of collaboration on safety of car 1 if car 1 decides to pass. It was observed that the controller of car 1 made the correct decision whether to execute the passing maneuver or not, based on the feasibility of the maneuver. When car 1 decided to pass, the collaboration between the autonomous vehicle and the human-driven vehicles reduced the crashes if car 1's decision was erroneous due to measurement noise. The work demonstrates how a Model-Based Systems Engineering approach can be used in the context of the car passing problem to manage complexity, developing virtual prototypes for analysis and deriving design requirements for autonomous vehicles.

**Keywords** — Car Passing (Overtaking) Problem, Model-Based Systems Engineering, SysML, Human and Autonomous Vehicle Collaboration, Measurement-based Decision-making Strategy

## I. INTRODUCTION

Today, autonomous vehicles are the “next big thing” in the automotive industry. Autonomous vehicles promise significant decrease in crashes by 90% [1], reduction in driving time by addressing the human elements of dynamic driving conditions, like drunken driving and distracted driving. One of the key maneuvers required to achieve these benefits is allowing autonomous vehicles to pass other vehicles in traffic. The passing maneuver is especially challenging when other vehicles are driven by humans, making the decision to pass prone to elements of human behavior (whether vehicles will collaborate or not). Therefore, as one of the most challenging driving maneuvers, passing maneuver is the focus of this work.

Studies on the passing behavior are focused on a broad variety of objectives. Some of the main recent studies are mentioned here. Liu and Wang (2017) have tried to predict the over-taking rate/zone in the opposite direction at overtaking

zones using traffic and geometric factors [2]. Ghods et al. (2016) built a model to study microscopic gap acceptance models as a function of each driver's perception of the expected time-to-collision (TTC) [3]. Llorca et al. (2015) focused on the development of a microscopic model of passing maneuver that incorporated the effect of factors such as available sight distance, delay and remaining travel time until the end of the highway segment [4]. Bahram et al. (2016) presented an online-capable Model-Based interaction-aware intention estimation with maneuver-based motion prediction based on supervised learning for dynamic environments [5]. Jarnea et al. (2015) presented a personal design of a driver assistance system for an overtake maneuver on a highway approach [6]. Mwesige et al. (2016) offered a regression model to predict the probability to end the passing maneuver with time-to-collision (TTC) less than 2 or 3 s gap [7]. Mihaly et al. (2015) modeled a highway overtaking scenario and performed simulations using the validated vehicle simulator vehicleSim [8]. Moreno et al. (2015) developed a design and marking criteria for minimum overtaking zone lengths, with traffic operational efficiency and safety taken into consideration [9]. Chen et al. (2015) developed and validated an algorithm that would aid in improving the overall effectiveness of forward collision warning (FCW) systems in overtaking maneuvers [10]. Chu et al. (2016) evaluated the effectiveness of a dedicated short-range communication (DSRC)-based wireless vehicle-to-vehicle (V2V) communication system devised for improving safety during overtaking maneuvers [11]. It can be seen that most of the work on passing behavior look at it from very different points of view, and all of them focus on human-driven cars. However, the focus of this work is on the AV as the primary vehicle performing the passing maneuver. A rigorous Model-Based system engineering approach has been used to analyze the effect of collaboration of human-driven cars on the design parameters of AV which is another novel aspect of this work.

In this paper, the primary goal is to study the effect of collaboration among drivers to mitigate the effects of measurement accuracy. This study continues with detailed explanations on the problem statement in section 2 and the design of the SysML models in section 3. The mathematical model is explained in section 4 and the discussion and results is presented in section 5. At the end, the conclusion is stated in section 6.

## II. PROBLEM STATEMENT

### A. Modeling Objective

The objective of modelling the car passing problem is to study the effects of collaboration on: 1) The correctness of the decision of whether to pass or not, made by car 1, and 2) The trajectory taken by car 1 to execute the passing maneuver. Collaboration means that cars 2 and 3 will deaccelerate to support the passing behavior after they infer that car 1 is executing the passing maneuver.

Car 1 must decide whether to pass or not and if it decides to pass it must safely pass car 2 in oncoming traffic. Safety of cars means that the distance between a car and its adjacent cars in the same lane at any time should be greater than a certain distance, which is called safety distance, 'sd'. This safety distance is used to check for collision possibilities. The decision to pass means finding a feasible trajectory that car 1 can follow to safely execute the passing maneuver. The decision to pass is based on noisy measurements. As measurement noise increases, correctness of the decision may decrease, leading to a crash. Collaboration of other cars can help reduce crashes while car 1 is performing the passing maneuver and make up for uncertainty in measurements. The three cars with their interactions have been modeled by developing a functional architecture, structure and behavior models, in SysML. This is followed by the development of a mathematical model in MATLAB to study the car passing problem.

### B. System of Interest

The system of interest consists of the three cars moving on a road as shown in Fig. 1. This interaction of the three cars and the road to achieve the objective of car 1 to safely pass car 2 in oncoming traffic is captured in the use case diagram in Fig. 2. To achieve this objective, car 1 must check the feasibility of the passing maneuver and if the maneuver is feasible, a reference trajectory is generated for car 1 to follow and safely pass the slower car in front of it (car 2).

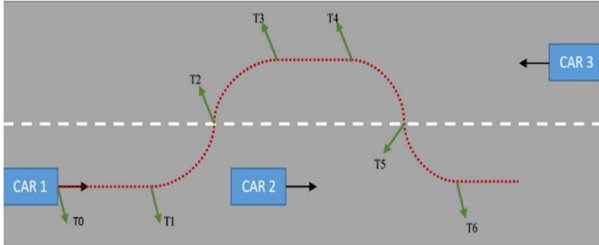


Fig. 1: System at  $t=0$  seconds with reference trajectory.

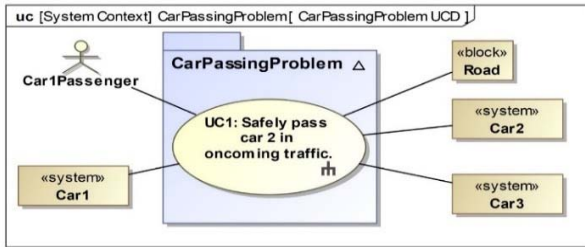


Fig. 2: Use Case Diagram for the Car Passing Problem.

- Phase 1: Car 1 is determining safety of passing maneuver.
- Phase 2: Car 1 determines it is safe and executes turning.
- Phase 3: Car 1 is entering the adjacent lane.

- Phase 4: Car 1 is trying to pass car 2.
- Phase 5: Car 1 starts turning after passing car 2.

Phase 6: Car 1 is moving back in the original lane.  $T_i$  and  $T_{i+1}$  represent starting and ending time points of phase ' $i+1$ ', where  $i \in \{1,2,3,4,5,6\}$ . ' $dTi$ ' is the time spent in phase ' $i$ ', where  $i \in \{1,2,3,4,5,6\}$ . The collaboration of the cars is defined as a Boolean variable, ' $Collaborative_j$ ' where  $j \in \{2,3\}$ , with value '1' referring to a cooperative driver and value '0' referring to a neutral driver. A cooperative driver will slow down to support the passing maneuver while a neutral driver will continue at its current velocity. It is assumed that collaboration starts at  $T3$ .

The decision to pass means finding a feasible trajectory that car 1 can follow to safely execute the passing maneuver. This decision primarily depends on:

- 1) Initial dynamics and positions:
  - $v_i(T0)$ : velocity of car ' $i$ ',
  - $a_i(T0)$ : acceleration/deacceleration of car ' $i$ ',
  - $w_1(T0)$ : angular velocity of car 1,
  - $x_i(T0)$ : x position of car ' $i$ ',
  - $y_i(T0)$ : y position of car ' $i$ ',
  - $\theta_1(T0)$ : angular orientation of car 1 w.r.t. x-axis.
- 2) Safety distance ' $sd_1$ ' used for trajectory planning by car 1,
- 3) Maximum speed ( $v_{max} = 115\text{km/hr}$ ) and minimum speed ( $v_{min} = 70\text{km/hr}$ ) allowed on the road,
- 4) Lane width ( $lw = 3.7\text{m}$ ),
- 5) Assumed angular orientation of car 1 at  $T2$  ( $\theta_1 = 30$  degrees) and  $T5$  ( $\theta_2 = -30$  degrees), and
- 6) Maximum acceleration of car 1 ( $a_{1max} = 2.77\text{m/s}^2$ ).

Assumed angular orientation of car 1 is the orientation of car 1 at  $T2$  used for trajectory planning in phase 1 and orientation of car 1 at  $T5$  used for trajectory planning in phases 1 through 4. The speed limits on the road and lane width are based on Maryland State Highways [12]. If car 1 decides to pass, execution of the passing maneuver is characterized by  $a_1$ ,  $w_1$  and  $dTi$ , where  $i \in \{1,2,3,4,5,6\}$ .

### C. Scope of the Paper

In this work, it is assumed that if at two consecutive time steps from  $T0$  through  $T1$ , car 1 finds out that it is infeasible to pass, only then passing maneuver is not executed. At  $T1$ , if the passing maneuver is feasible, car 1 must execute the passing maneuver and cannot go back. Cars 2 and 3 always move straight and have constant velocities from  $T0$  through  $T3$ . Moreover, velocity of car 2 is less than the velocity of car 1 at  $T0$ , so that passing behavior sounds reasonable. Position and velocity of cars along x and y axis shown in Fig. 3 is positive.  $\theta_1$  and  $w_1$  are positive in clockwise direction.

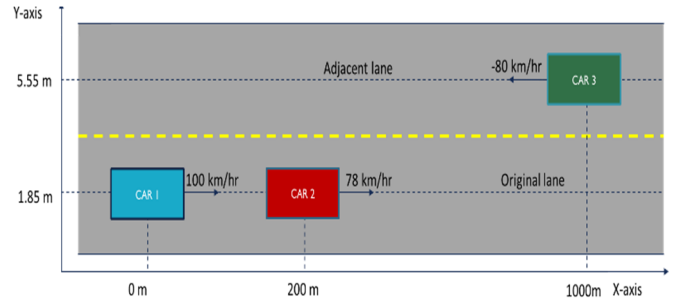


Fig. 3: Initial State of the System.

All the cars are modelled as point masses, with  $sd$  and  $sd_i$  (chosen to be greater than the length of the cars) are used for trajectory planning. The chosen values are:  $sd = 5m$  and  $sd_i = 10m$ , where  $i \in \{1,2,3\}$ . Additionally, the three cars are assumed to be of same make, i.e.,  $a_{1max} = a_{2max} = a_{3max} = 2.77m/s^2$  and  $d_{1max} = d_{2max} = d_{3max} = 10m/s^2$  [13], where  $d_{imax}$  is the maximum deceleration of car  $i$ . Also, all the cars strictly follow the speed limits on the road. The initial state of the system used for analyses in the work is shown in Fig. 3.

### III. SYSML MODEL

#### A. Structure Model

The car passing problem model consists of a system of interest; 'Car1', its user; 'Car1Passenger' and its environment. Environment consists of models of the external system; 'Car2', 'Car3' and 'Road' as shown in Fig. 4. Further, each block in the figure contains the parameters, use cases, behaviors and ports assigned to the model of that block. Car 1 is characterized by  $a_{1max}$  and  $d_{1max}$ . Cars 2 and 3 are characterized by  $(a_{2max}, d_{2max})$  and  $(a_{3max}, d_{3max})$  respectively. Each car model is composed of a measurement system, a controller and an actuator. Structure of car 1 and car 2 model are shown in Fig. 5 and Fig. 6 respectively. Structure of car 3 model is like the one of car 2, as such, is not shown here.

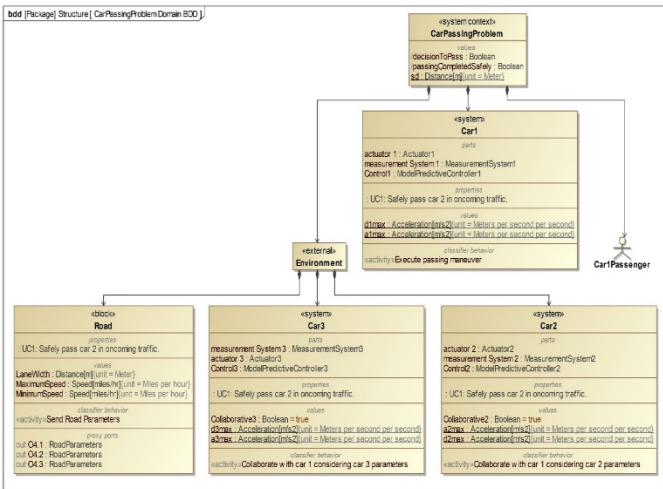


Fig. 4: Car Passing Problem Domain

Measurement system of car 1 replicates the noisy measurements taken by different sensors of car 1. The noise in measurements is modelled with white Gaussian noise with mean noise being '0'. The standard deviation of this noise is represented as a percentage of the measurement taken by the sensor where  $SDi_{posn}$  is standard deviation for position measurement for car  $i$ ,  $SDi_{vel}$  is standard deviation for velocity measurement for car  $i$ , and  $SDi_{accln}$  is standard deviation for acceleration measurement for car  $i$ . A subscript of 'self' denotes standard deviation for measurements about the car itself. The measurements are the x and y positions, velocities, acceleration, angular orientation and angular velocity of cars. All the measurements are relative measurements taken at intervals of  $dtM1$  seconds.  $dtM1$  represents measurement delay of the sensors. Measurement systems of cars 2 and 3 represent the perception of drivers of cars 2 and 3. Uncertainty in the perceived dynamics of cars 2 and 3 by the drivers is modelled

the same way as car 1 with reaction time of drivers being  $dtM2$  seconds and  $dtM3$  seconds respectively. The controller of each car is characterized by the control action, i.e.,  $(a_i, w_i)$  for car 1, acceleration  $a_2$  for car 2, acceleration  $a_3$  for car 3, and safe distance,  $sd_i$ , used for trajectory planning by car  $i$ . For cars 2 and 3, the safe distance  $sd_i$  represents the safe distance that the drivers of these cars want to maintain with the other car. Controller of car 1 is further characterized by:

- Decision to pass:  $'isPassingNotPossible'$  or  $'isPassingPossible'$ ,
- Whether passing maneuver is complete or not:  $'isPassingComplete'$ ,
- $Car1TrajAngle1: \theta_1$ ,
- $Car2TrajAngle2: \theta_2$ ,
- The clearance distance of car 1 w.r.t. car 2 at  $T5$  used for trajectory planning:  $'sd_{125}'$ ,
- And the clearance distance of car 1 w.r.t. car 3 at  $T5$  used for trajectory planning:  $'sd_{135}'$ .

The controllers of cars 2 and 3 are also characterized by  $\theta_2$ ;  $'Car2TrajAngle2'$ . It is assumed that cars 2 and 3 know  $\theta_2$ . Actuators of all cars are characterized by the dynamics of the car  $i$  at current time step;  $(x_{iR}, y_{iR}, v_{iR}, a_{iR}, w_{iR}, \theta_{iR})$  and previous time step;  $(x_{i0}, y_{i0}, v_{i0}, a_{i0}, w_{i0}, \theta_{i0})$ . These dynamics are free of noise.

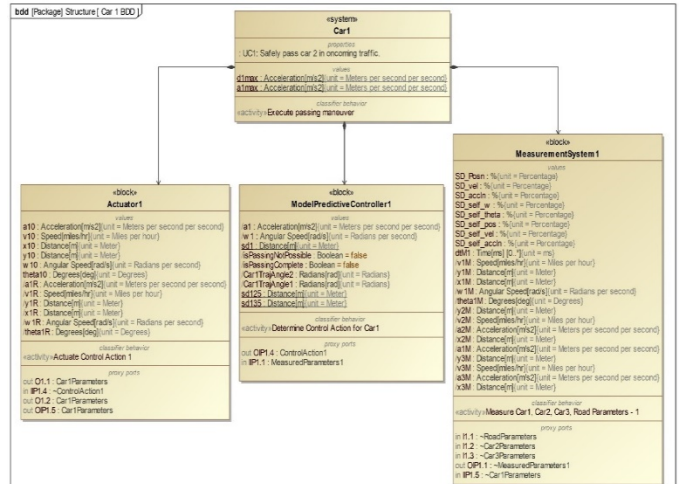


Fig. 5: Car 1 Block Definition Diagram

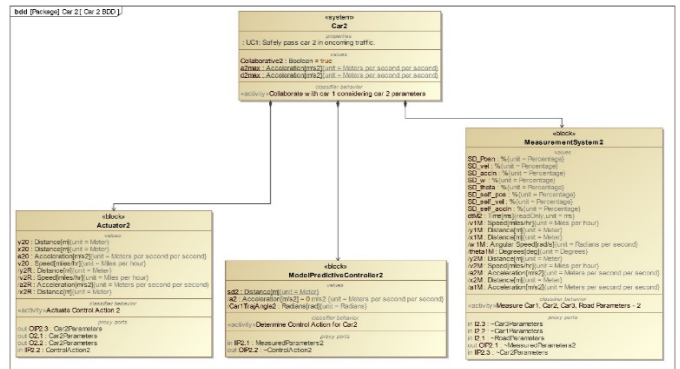


Fig. 6: Car 2 Block Definition Diagram

The measurement system of each car has interfaces with actuators of all other cars as shown in Fig. 7. The information flow and direction across these interfaces is defined in the Fig.

8. The direction for each interface is also shown in the corresponding ports in Fig. 7. These ports are also indicated in the Fig. 4, Fig. 5 and Fig. 6. The measurement system of each car has interface with the controller of the car. The controller of each car has interface with the actuator of that car. The actuator of the car has further interface with the measurements system of that car. The internal structure of car 1 is shown in Fig. 9. The internal structure of cars 2 and 3 are like the one for car 1. The information flow and direction across these interfaces is defined in the Fig. 10 for cars 1 and 2. Information flow for car 3 is like car 2. The diagrams shown in Fig. 7 through Fig. 10 are used to integrate the model for car passing problem using the sub models for each component.

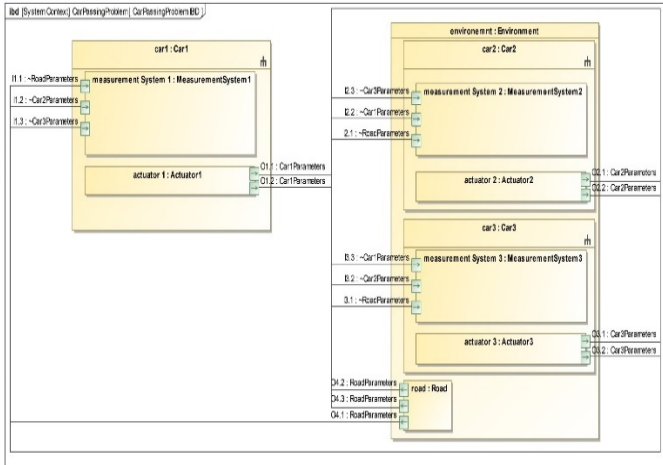


Fig. 7: Car Passing Problem Internal Block Diagram

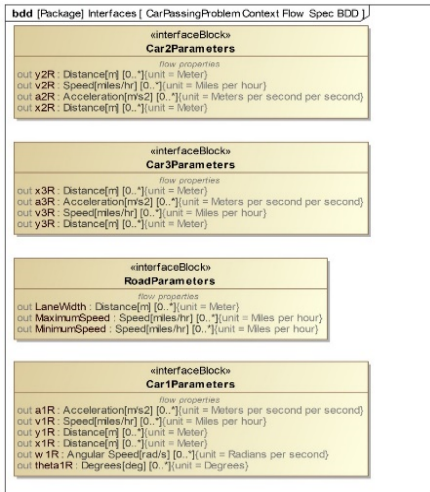


Fig. 8: Definition of Interfaces of the Model of Car Passing Problem

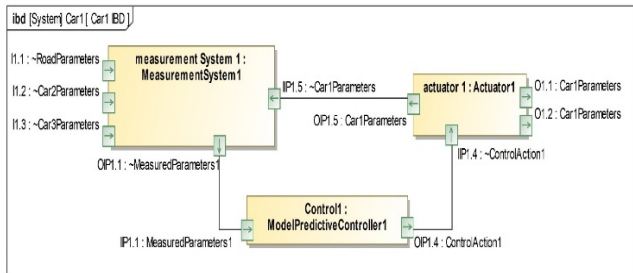


Fig. 9: Internal Block Diagram of Car 1

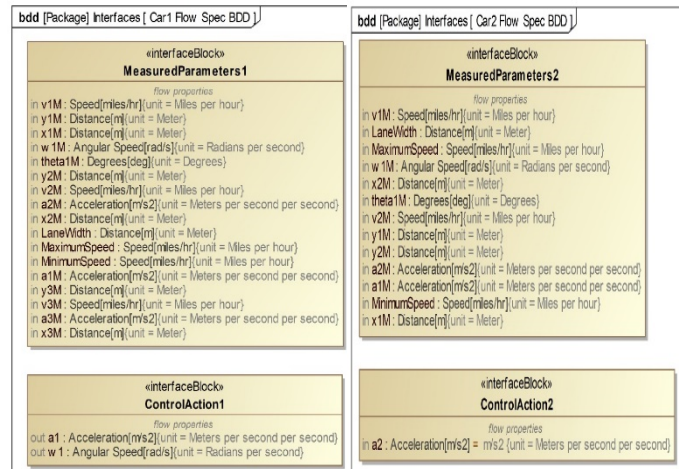


Fig. 10: Definition of Interfaces of Car 1 and Car 2

### B. Behavior Model

The behavior models are used to describe how the models of different components of the car passing problem function, respond to different inputs and interact with each other. This is done through mapping control flows (dashed arrow lines) and data flows (solid arrow lines) within and across different components of car passing problem. The high-level mapping of behavior for all the cars and road is shown through a context-level activity diagram in Fig. 11. The ports on the actions correspond to the ports shown in internal block diagrams. The diagram shows that the models of all four components run independent of each other with behavior allocated to the cars being executed every  $dtM_i$  seconds. Execution of all the components contained in the car passing problem starts simultaneously while all the cars have different conditions when the simulation ends. The simulation for car passing problem ends either if:

- Car 1 decides not to pass from  $T0$  to  $T1$ , or
- It successfully completes the passing maneuver, or
- It crashes.

This along with the control flow from one state of the car passing problem to another state is shown in the state machine diagram in Fig. 12. All the guard conditions are based on accurate states of all the cars. For car 1, simulation ends when car 1 either decides not to pass from  $T0$  to  $T1$ , it successfully completes the passing maneuver, or it crashes. Car 1's crash behavior is allocated to car passing problem as the crash is checked outside the model of car 1. For car 2, simulation ends when car 2 determines car 1 has completed the passing maneuver. For car 3, simulation ends at  $T5$ . This is shown in Fig. 11.

The behaviors allocated to the models of the three cars in Fig. 11 are further decomposed to define actions that are executed in a sequence to achieve the functionality associated with context-level behavior. These actions are then allocated to the components of the car model. The activity diagram for the 'Execute passing maneuver' allocated to car 1 is shown in Fig. 13. The frame of the activity diagram contains inputs to and outputs from the behavior. Similar activity diagrams were also drawn for the other two cars with almost similar behavioral

decomposition. The measurement system of each car takes in measurements periodically from other cars and sends noisy measurements to the controller. The controller uses these measurements to determine a control action for the car. The actuator receives that control action and actuates it. The updated state of the dynamics of the car is then available to the measurement systems of other cars.

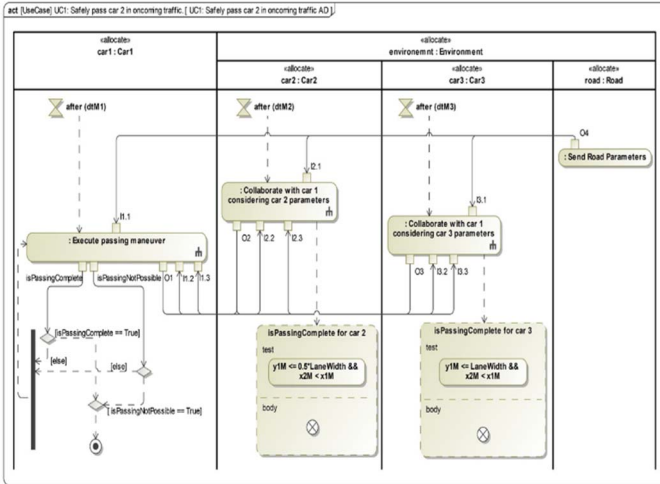


Fig. 11: Context-level activity diagram of the Car Passing Problem Model

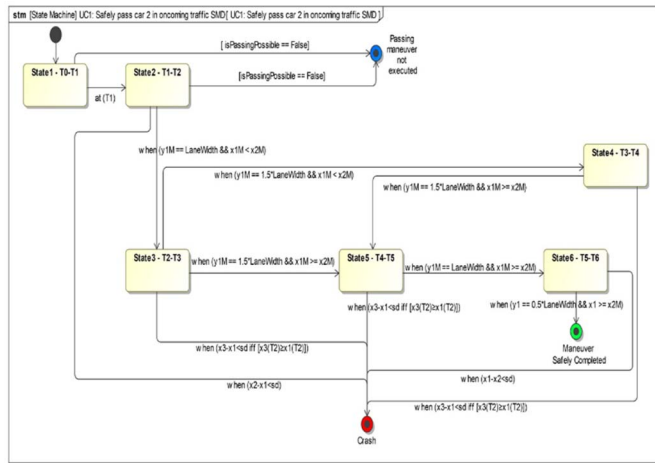


Fig. 12: State Machine diagram for the Car Passing Problem Model

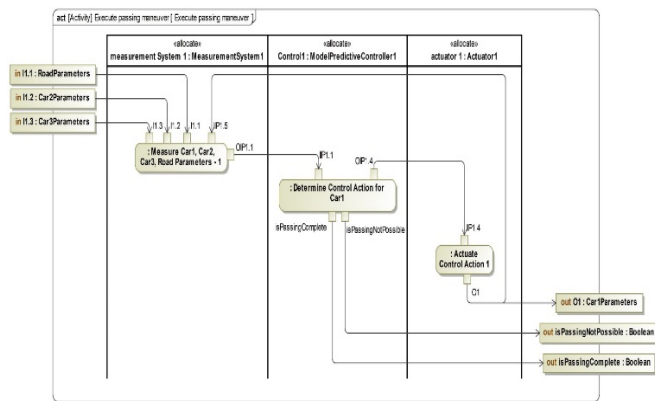


Fig. 13: Activity Diagram for Car 1's 'Execute Passing Maneuver' Behavior

The control action determination behavior allocated to the controller has been further explained by using state machine diagrams. The state machine diagram for the controller of car 1

is shown in Fig. 14. The diagram identifies the different guard conditions and states that car 1 should take to reach a desired end state – either decide not to pass or pass safely. State ‘i’ in the diagram corresponds to the phase ‘i’ of the trajectory of car 1. In states 1 and 2, feasibility of the passing maneuver is checked followed by determination and dissemination of control signal. In states 3 through 6, determination of the reference trajectory is followed by dissemination of the control signal. The frequency of each composite state, their internal states and entry and exit guard conditions are shown in the state machine diagram. Here, all the guard conditions are based on the measured values of the states of the cars as only the measured parameters of the cars are available to the controller.

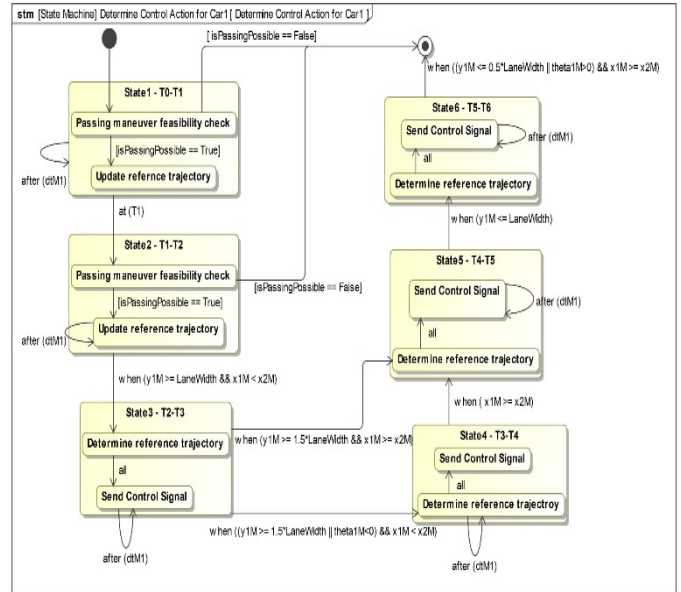


Fig. 14: State Machine Diagram for Controller of Car 1

The state machine diagram for the controller of car 2 is shown in Fig. 15. It has two states. One thing to note is that these states do not correspond to a single phase of car 1's trajectory. State 1 corresponds to phases 1 through 3, where car 2's acceleration is  $0m/s^2$ . If the driver of car 2 is non-collaborative in phases 4 through 6, car 2 remains in state 1 and its acceleration is  $0m/s^2$ . State 2 corresponds to phases 4 through 6, where car 2 decelerates to collaborate with car 1. To collaborate, car 2 first uses the available measurements to predict the trajectory of car 1. This is done by determining which phase of the trajectory car 1 is in. As the shape of the trajectory is assumed to be fixed, and car 2 has information about the shape of the trajectory and behavior of car 1, it uses relevant kinematic equations for prediction. This is done in internal state, State 2/Phase ‘i’. The predicted position of car 1 at  $T5$  is used to determine the control action. A similar state machine diagram for the controller of car 3 was drawn. The difference is that the controller of car 2 uses measurements about car 1 to predict while the controller of car 3 uses measurements about both cars 1 and 2 to predict. This is because the planning of the trajectory of car 1, like different turning times, is mostly based on car 2. Also, as the simulation of car 3 ends when car 1 has entered the original lane, i.e.  $T5$ , the state machine diagram for the controller of car 3 does not

have the internal state - State 2/Phase 5. Further, the model of car 3 does not consider parameters of car 2 because car 2 is in other lane throughout the simulation and does not directly interact with car 2.

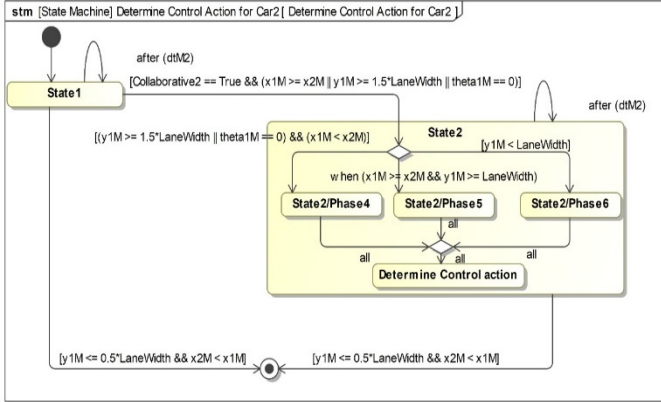


Fig. 15: State Machine Diagram for Controller of Car 2

#### IV. MATHEMATICAL MODEL

The structure and behavior of the model of the car passing problem discussed in section III was further accentuated by a mathematical model for the controller of the three cars. The purpose of the controller is to model the decision-making strategy and reference trajectory for car 1, and trajectories of cars 2 and 3. For car 1, the reference trajectory is the optimal solution to the problem with the objective to maximize distance between car 1 and car 2 at  $T_5$  if car 1 is in phase 1 through 5 of the trajectory. If the car is in phase 6, the objective is to maximize the distance between car 1 and car 2 at  $T_6$ . This is shown in the objective function in (1). Here,  $i = 5$  if it is in phases 1 through 5 and  $i = 6$  if it is in phase 6. Further, any such trajectory should ensure that car 1 maintains a safe distance ' $sd_i$ ' with respect to the other cars at the end of each phase of, indicated in (2) – (4) and (6) – (9). Equations (5) and (10) are used in place of (4) and (9) respectively in phases 1 and 2 of the trajectory while car 1 is making the decision to pass or not. As car 1 cannot decelerate and cars 2 and 3 cannot accelerate, ensuring the safe distance is maintained at the end points of each phase, it is guaranteed that safe distance is maintained throughout the phase as well. Then, there are constraints based on the maximum and minimum speed limits on road (11), maximum acceleration of car 1 (12) and the definition of the starting and ending points of the phases (13) – (17). These constraints along with the objective function form the mathematical model for car 1.

$$\max. [x_1(T_i) - x_2(T_i)]^2 \quad (1)$$

$$x_2(T_1) - x_1(T_1) \geq sd_1 \quad (2)$$

$$x_2(T_2) - x_1(T_2) \geq sd_1 \quad (3)$$

$$x_1(T_5) - x_2(T_5) \geq sd_1 \quad (4)$$

$$x_1(T_5) - x_2(T_5) \geq sd_{125} \quad (5)$$

$$x_1(T_6) - x_2(T_6) \geq sd_1 \quad (6)$$

$$x_3(T_3) - x_1(T_3) \geq sd_1 \quad (7)$$

$$x_3(T_4) - x_1(T_4) \geq sd_1 \quad (8)$$

$$x_3(T_5) - x_1(T_5) \geq sd_1 \quad (9)$$

$$x_3(T_5) - x_1(T_5) \geq sd_{135} \quad (10)$$

$$v_{min} \leq v_1(t) \leq v_{max}, \text{ where } t \in [T_0, T_6] \quad (11)$$

$$0 \leq a_1(t) \leq a_{max}, \text{ where } t \in [T_0, T_6] \quad (12)$$

$$y_1(T_2) = lw \quad (13)$$

$$y_1(T_3) = 1.5 * lw \quad (14)$$

$$x_1(T_4) = x_2(T_4) \quad (15)$$

$$y_1(T_5) = lw \quad (16)$$

$$y_1(T_6) = 0.5 * lw \quad (17)$$

Cars 2 and 3 are driven by human drivers. A human driver generally uses the perceived information about the other cars to make an estimate about the future state of the cars and determine a generic trajectory they could take. Then, the human driver decides what action to take. This basic process is embodied in the design of controller for car 2. The controller uses the measured parameters of car 1 to determine the future trajectory of the car 1. It is assumed that collaboration starts at  $T_3$  and car 2 knows about the shape of car 1's trajectory. So, it uses (18) – (21) to determine  $T_4$ ,  $T_5$  and  $T_6$ . This is followed by determining the position of car 1 at these time instants based on the most recent available measurements about cars 1 and 2. Next, the controller chooses the minimum deceleration to maintain a safe distance, ' $sd_2$ ', with car 1 at  $T_5$  and  $T_6$  (21). This is done by satisfying (22) – (24). Then, there are constraints based on the maximum and minimum speed limits on the road (25) and the maximum acceleration of car 2 (26). These constraints along with the objective function form the mathematical model for car 2.

$$x_1(T_4) = x_2(T_4) \quad (18)$$

$$y_1(T_5) = lw \quad (19)$$

$$y_1(T_6) = 0.5 * lw \quad (20)$$

$$\text{minimize } (a_2)^2 \quad (21)$$

$$x_1(T_5) - x_2(T_5) \geq sd_2 \quad (22)$$

$$x_1(T_6) - x_2(T_6) \geq sd_2 \quad (23)$$

$$v_{1x}(T_5) \geq v_2(T_5) \quad (24)$$

$$v_{min} \leq v_2(t) \leq v_{max}, \text{ where } t \in [T_3, T_6] \quad (25)$$

$$0 \geq a_2(t) \geq -d_{2max}, \text{ where } t \in [T_3, T_6] \quad (26)$$

The mathematical model of car 3 is very similar to car 2. The only difference is that car 3 makes estimates about the trajectory of car 1 based on measurements about both cars 1 and 2. Also, there are no constraints on car 3 after  $T_5$  as car 3 interacts with car 1 till  $T_5$  only. The relevant equations are from (27) – (32).

$$x_1(T_4) = x_2(T_4) \quad (27)$$

$$y_1(T_5) = lw \quad (28)$$

$$\text{minimize } (a_3)^2 \quad (29)$$

$$x_3(T_5) - x_1(T_5) \geq sd_3 \quad (30)$$

$$-v_{max} \leq v_3(t) \leq -v_{min}, \text{ where } t \in [T_3, T_5] \quad (31)$$

$$0 \leq a_3(t) \leq d_{3max}, \text{ where } t \in [T_3, T_5] \quad (32)$$

## V. DISCUSSION AND RESULTS

### A. Integration of the models and Execution

This section explains how the SysML model aided in developing an executable model of the car passing problem in MATLAB. The block definition diagrams were used to identify and develop the modules of different components and declare variables in the MATLAB. This was followed by using internal block diagrams to define how different modules are connected to each other and what information can flow across them during simulation execution. Activity diagrams were used to define the sequence of control and data flow across the individual modules and overall model to achieve the assigned functionality. This was followed by using the state machine diagrams to develop guard conditions to determine how and when a module or part of a module is called during model execution in the MATLAB. Lastly, the control action for each car was achieved by implementing the mathematical model and solving optimization problem using ‘fmincon’ in MATLAB.

### B. Effects of Collaboration

This section represents the effects of collaboration amongst the drivers on safety of the passing maneuver. To study these effects, an initial state of the system was chosen for which passing was not possible, refer to Fig. 3, when there is no collaboration. Acceleration of car 1 is assumed to be  $1m/s^2$ . Acceleration of cars 2 and 3 is  $0m/s^2$ . Table 1 lists the values of different parameters of the car passing problem. Standard deviation of noise in position and velocity measurement for car 1 is based on [14] and [15]. It is based on [16] and [17] for cars 2 and 3. ‘dtM1’ and ‘dtM2’/‘dtM3’ are based on [18] and [19].

Table 1: Operational parameters of the Model

$sd_1$	10 m	$sd_3$	10 m	$SD_{2posn}$	8.6%
$sd_2$	10 m	$sd_{125}$	35 m	$SD_{2vel}$	10%
$sd_{135}$	35 m	$sd$	5 m	$SD_{2accln}$	5%
$SD_{1posn}$	2.5%	$SD_{3posn}$	8.6%	$dtM1$	0.1 s
$SD_{1vel}$	2.91%	$SD_{3vel}$	10%	$dtM2$	0.9 s
$SD_{1accln}$	2.5%	$SD_{3accln}$	5%	$dtM3$	0.9 s

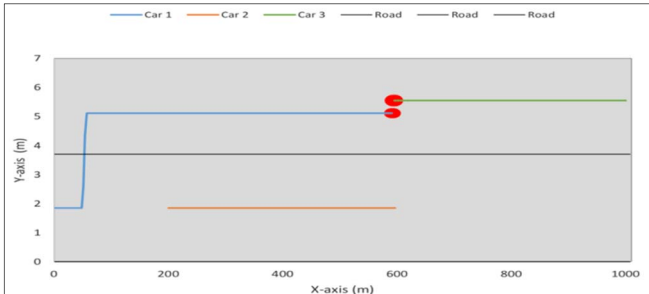


Fig. 16: Car 1 crashing with Car 3

Further, the passing maneuver was executed till either there was a crash, or the passing maneuver was safely completed. The corresponding trajectory is shown in Fig. 16 where car 1 crashes with car 3 in phase 4 of the trajectory. As car 2 collaborates

with car 1 and slows down after T3, car 1 spends less time in phase 4 of the trajectory and can safely complete the passing maneuver as shown in Fig. 17. When car 3 collaborates with car 1 and slows down after T3, car 1 can safely complete the passing maneuver, as shown in Fig. 18. When both cars 2 and 3 collaborate, car 1 safely completes the passing maneuver by travelling less distance, as shown in Fig. 19.

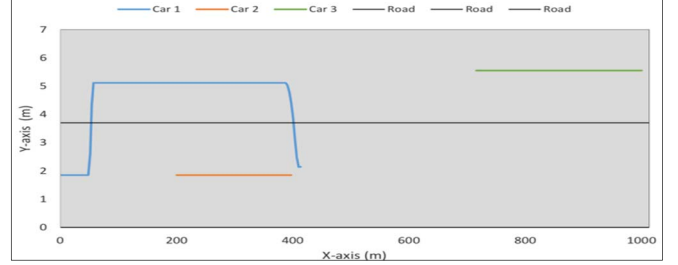


Fig. 17: No crash due to collaboration of Car 2

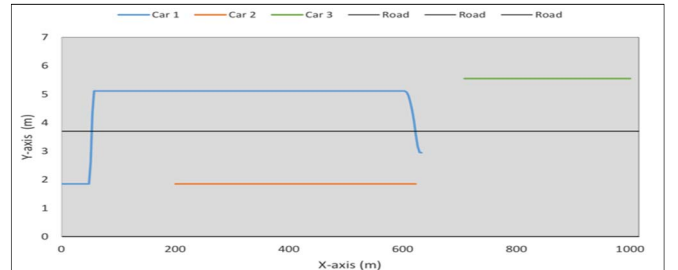


Fig. 18: No crash due to collaboration of Car 3

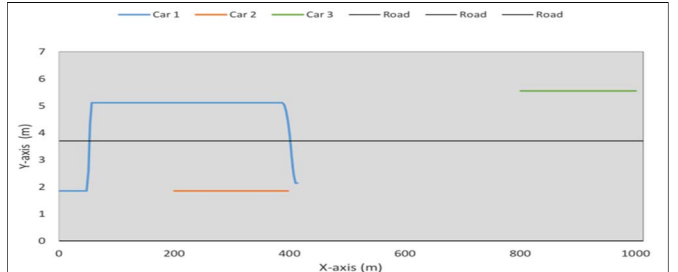


Fig. 19: Car 1 passing due to collaboration with Car 2 and Car 3

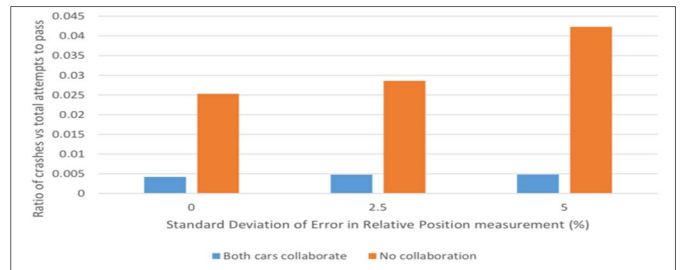


Fig. 20: Effect of collaboration on Relative Position measurement

To study the effects of collaboration, 1000 iterations of simulation were done. For the cases where car 1 decided to pass, they were segregated into 2 classes - where both collaborated, and the case where either of the cars or none of the cars collaborated. For each class, the number of crashes was counted and divided by the total count of instances in that class to get the ratio of crashed versus total count of attempts to pass. Fig. 20 to Fig. 22 show the effect of collaboration and standard deviation of error on relative position, relative velocity and relative acceleration measurement (%) on safety. The

simulation provides expected results as shown in these figures; collaboration significantly decreases the count of crashes for high uncertainty in position, velocity and acceleration measurement. This validates the simulation. This shows that the collaboration among the three cars, can help to mitigate the uncertainty in measurements.

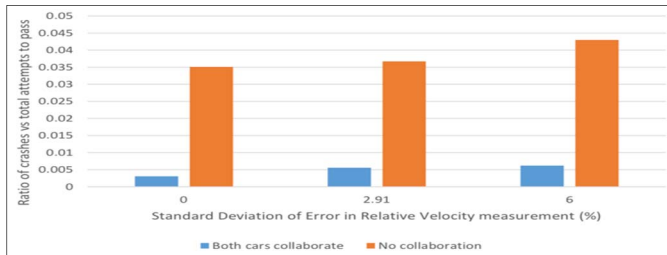


Fig. 21: Effect of collaboration on Relative Velocity measurement

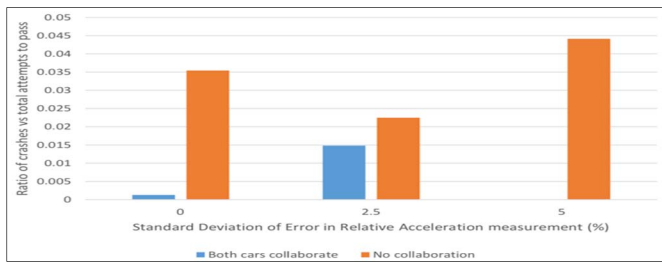


Fig. 22: Effect of collaboration on Relative Acceleration measurement

## VI. CONCLUSIONS

The significant purpose of this work was to develop a framework to evaluate measurement-based decision-making strategies for the AV satisfying driving safety constraints. The problem was structured using SysML to build a modular architecture with clearly defined interfaces and allocated behaviors. The functional architecture of the car passing problem was modelled using rigorous Model-Based Systems Engineering and the mathematical model of different components was developed in MATLAB using the functional architecture. The performance of the system in terms of the safety of the passing maneuver was evaluated and it was shown that the collaboration among the three cars, can help to mitigate the uncertainty in measurements. The work demonstrated how Model-Based Systems Engineering approach can be used in the context of the car passing problem to facilitate managing complexity, making system modular while allowing design space exploration and evaluation. Better results can be obtained by conducting human factors studies to relate cooperativeness of the drivers towards other vehicles passing them and how they react to the situation. One must always consider that the car passing problem is critical to human safety and the Model-Based Systems Engineering approach will help towards developing safer and more reliable systems.

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