

Timed Automata Approach for Motion Planning Using Metric Interval Temporal Logic

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Abstract—In this paper, we consider the robot motion (or task) planning problem under some given bounded time high level specifications. We use metric interval temporal logic (MITL), a member of the temporal logic family, to represent the task specification and then we provide a constructive way to generate a timed automaton and methods to look for accepting runs on the automaton to find a feasible motion (or path) sequence for the robot to complete the task.

Index Terms—Timed automata, Temporal Logic, Metric Temporal Logic

I. INTRODUCTION

Motion planning and task planning have gained an enormous thrust in the robotics community in the past decade or so. Although motion (task) planning has attracted a great deal of research in the past few decades, recently, researchers have come up with new metrics and methodologies to represent motion and task specifications. Initially, motion planning for a mobile robot started with the aim of moving a point mass from an initial position to a final position in some optimal fashion. Through the course of time, people started to consider planning in cluttered domains (i.e. in presence of obstacles) and also accounted for the dimensionality and the physical constraints of the robot.

Though we have efficient approaches for general motion planning, very few are available or scalable to plan in dynamic environments or under finite time constraints. Temporal logics have been used greatly to address complex motion specifications, motion sequencing and timing behaviors etc. Historically temporal logic was originated for model checking, validation and verification in the software community [1] and later on researchers found it very helpful to use Linear Temporal logic (LTL), Computational Tree logic (CTL), Signal Temporal logic (STL) etc. for representing complex motion (or task) specifications. The developments of tools such as SPIN [2], NuSMV [3] made it easier to check if given specifications can be met by creating a suitable automaton and looking for a feasible path for that automaton. However, the construction of the automaton from a given temporal logic formula is based on the implicit assumption that there is no time constraints associated with the specification.

Currently motion planning for robots is in such a stage where it is very crucial to incorporate time constraints since

these constraints can arise from different aspects of the problem: dynamic environment, sequential processing, time optimality etc. Planning with time bounded objectives is inherently hard due to the fact that every transition from one state to another in the associated automaton has to be carried out, by some controller, exactly in time from an initial configuration to the final configuration. Time bounded motion planning has been done in heuristic ways [4], [5] and also by using a mixed integer linear programming (MILP) framework [6], [7]. In this paper, we are interested in extending the idea of using LTL for time-unconstrained planning to use MITL for time-constrained motion planning. In [8], the authors proposed a method to represent time constrained planning task as an LTL formula rather than MITL formula. This formulation reduced the complexity of EXP-SPACE-complete for MITL to PSPACE-complete for LTL. However, the number of states in the generated Büchi automata increases with the number of steps.

In this paper, we mainly focus on motion planning based on the construction of an efficient timed automaton from a given MITL specification. A dedicated controller to navigate the robot can be constructed for the general planning problem once the discrete path is obtained from the automaton. The earlier results on construction of algorithms to verify timing properties of real time systems can be found in [9]. The complexity of satisfiability and model checking problems for MTL formulas has been already studied in [10] and it has been shown that commonly used real-time properties such as bounded response and invariance can be decided in polynomial time or exponential space. More works on the decidability on MTL can be found in [11] and the references there in. The concept of alternating timed automata for bounded time model checking can be found in [12]. [13] describes the construction of deterministic timed automata from MTL specifications and this provides a unified framework to include all the future operators of MTL. The key to the approach of [13] was in separating the continuous time monitoring from the discrete time predictions of the future. We restrict our attention to generate timed automata from MITL based on the work done in [14]. This is accomplished by constructing a timed automaton to generate a sequence of states and another one to check whether the sequence generated is actually a valid one in the sense that it satisfies the given MITL specification.

The rest of the paper is organized as follows, section II provides a background on MITL and the timed automata based approach for MITL. Section III illustrates how the timed automata can be used for motion synthesis and we also

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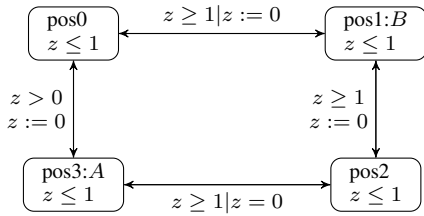


Fig. 1. Timed Automata based on cell decomposition and robot dynamics

provide UPPAAL [15] implementation of the same. Section IV gives some examples on different time bounded tasks and shows the implementation results. Section V provides a brief overview of how a continuous trajectory can be generated from the discrete plan. Finally, we conclude in section VI.

II. PRELIMINARIES

In this paper, we consider a surveying task in an area by a robot whose motion is abstracted to a graph. In particular for our particular setup, the robot motion is captured as a timed automaton (Fig. 1). Every edge is a timed transition that represents navigation of the robot from one location to another in space and every vertex of the graph represents a partition of the space. Our objective is to find an optimal time path that satisfies the specifications given by timed temporal logic.

A. Metric Interval Temporal Logic (MITL)

Metric interval temporal logic is a specification that includes timed temporal specifications for model checking. It differs from Linear Temporal Logic in the part that it has bounded time constraints on the temporal operators.

The formulas for LTL are build on atomic propositions by obeying the following grammar.

Definition 2.1: *The syntax of LTL formulas are defined according to the following grammar rules:*

$$\phi ::= \top \mid \pi \mid \neg\phi \mid \phi \vee \phi \mid \mathbf{X}\phi \mid \phi \mathbf{U}\phi$$

$\pi \in \Pi$ the set of propositions, \top and $\perp (= \neg\top)$ are the Boolean constants *true* and *false* respectively. \vee denotes the disjunction operator and \neg denotes the negation operator. \mathbf{U} represents the Until operator. MITL extends the Until operator to incorporate timing constraints.

Definition 2.2: *The syntax of MITL formulas are defined according to the following grammar rules:*

$$\phi ::= \top \mid \pi \mid \neg\phi \mid \phi \vee \phi \mid \phi \mathbf{U}_I \phi$$

where $I \subseteq [0, \infty]$ is an interval with end points in $\mathbb{N} \cup \{\infty\}$. \mathbf{U}_I symbolizes the timed Until operator. Sometimes we will represent $\mathbf{U}_{[0, \infty]}$ by \mathbf{U} . Other Boolean and temporal operators such as conjunction (\wedge), eventually within I (\diamond_I), always on I (\square_I) etc. can be represented using the grammar described in definition 2.2. For example, we can express time constrained eventually operator $\diamond_I \phi \equiv \top \mathbf{U}_I \phi$ and so on. In this paper all the untimed temporal logic specifications are transformed into expressions involving the Until operator and all the timed operator specifications are transformed to eventually within I , to make it easier to generate a timed automaton.

MITL is interpreted over n -dimensional Boolean ω -sequences of the form $\xi : \mathbb{N} \rightarrow \mathbb{B}^n$, where n is the number of propositions.

Definition 2.3: *The semantics of any MTL formula ϕ is recursively defined over a trajectory (ξ, t) as:*

$(\xi, t) \models \pi$ iff (ξ, t) satisfies π at time t

$(\xi, t) \models \neg\pi$ iff (ξ, t) does not satisfy π at time t

$(\xi, t) \models \phi_1 \vee \phi_2$ iff $(\xi, t) \models \phi_1$ or $(\xi, t) \models \phi_2$

$(\xi, t) \models \phi_1 \wedge \phi_2$ iff $(\xi, t) \models \phi_1$ and $(\xi, t) \models \phi_2$

$(\xi, t) \models \bigcirc\phi$ iff $(\xi, t+1) \models \phi$

$(\xi, t) \models \phi_1 \mathbf{U}_I \phi_2$ iff $\exists s \in I$ s.t. $(\xi, t+s) \models \phi_2$ and $\forall s' \leq s$, $(\xi, t+s') \models \phi_1$.

Thus, the expression $\phi_1 \mathbf{U}_I \phi_2$ means that ϕ_2 will be true within time interval I and until ϕ_2 becomes true, ϕ_1 must be true. The MITL operator $\bigcirc\phi$ means that the specification ϕ is true at the next time instance, $\square_I \phi$ means that ϕ is always true for the time duration I , $\diamond_I \phi$ means that ϕ will eventually become true within the time interval I . Composition of two or more MITL operators can express very sophisticated specifications; for example $\diamond_{I_1} \square_{I_2} \phi$ means that within time interval I_1 , ϕ will be true and from that instance it will hold true always for a duration of I_2 . Other Boolean operators such as implication (\Rightarrow) and equivalence (\Leftrightarrow) can be expressed using the grammar rules and semantics given in definitions 2.2 and 2.3. More details on MITL grammar and semantics can be found in [16], [9].

B. MITL and Timed Automata Based Approach

An LTL formula can be transformed into a Büchi automaton which can be used in optimal path synthesis [17] and automata based guidance [18]. Similarly, in this paper, we focus on developing a timed automata based approach for MITL based motion planning. MITL, a modification of Metric Temporal Logics (MTL), disallows the punctuation in the temporal interval, so that the left boundary and the right boundary have to be different. In general the complexity of model checking for MTL related logic is higher than that of LTL. The theoretical model checking complexity for LTL is PSPACE-complete [19]. The algorithm that has been implemented is exponential to the size of the formula. MTL by itself is undecidable. The model checking process of MITL includes transforming it into a timed automaton [9] [14]. CoFlatMTL and BoundedMTL defined in [20] are more expressive fragments of MTL than MITL, which can be translated to LTL-Past but with exponential increase in size. SafetyMTL [11] and MTL, evaluated over finite and discrete timed words, can be translated into alternative timed automata. Although theoretically, the results suggest many fragments of MTL are usable, many algorithms developed for model checking are based on language emptiness check, which are very different from the control synthesis i.e. finding a feasible path. To the best of our knowledge, the algorithm that is closer to implementation for motion planning is that of [14].

This paper uses the MITL and timed automaton generation based on [14]. In the following section, the summary of the

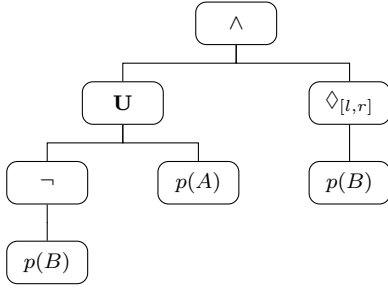


Fig. 2. Logic tree representation of ϕ .

transformation and our implementation for control synthesis are discussed.

III. MITL FOR MOTION PLANNING

A. MITL to Timed Automata Transformation

Consider the following requirements: a robot has to eventually visit an area A and another area B in time interval $[l, r]$, and the area A has to be visited first. This can be captured in the following MITL expression,

$$\phi = (\neg BUA) \wedge (\diamond_{[l,r]}B)$$

This expression can be represented by a logic tree structure, where every node that has children is a temporal logic operator and every leaf node is an atomic proposition, as shown in Fig. 2. Every link represents an input output relationship.

The authors in [14] propose to change every temporal logic operator into a timed signal transducer, which is a temporal automaton that accepts inputs and generates outputs. Based on the definitions in [14] the Timed Transducer (TT) used in this paper is defined in Definition 3.1 below, to fit the control synthesis problem,

Definition 3.1 (Timed Transducer): An timed transducer is a tuple $\mathcal{A} = (\Sigma, Q, \Gamma, \mathcal{C}, \lambda, \gamma, I, \Delta, q_0, F)$, where Σ is the input alphabet, Q is the finite set of discrete states, Γ is the output alphabet, \mathcal{C} is the set of clock variables, and I is the invariant condition defined by conjunction of inequalities of clock variables. The clock variables can be disabled and activated by setting the rate of the clock to 0 or 1 in the invariant I .

$\lambda : Q \rightarrow \Sigma$ is the input function, which labels every state to an input, while

$\gamma : Q \rightarrow \Gamma$ is the output function, which labels every state to an output.

Δ is the transition relationship between states which is defined by (p, q, r, g) , where p is the start state, q is the end state, r is the clock resets, and g is the guard condition on the clock variables.

q_0 is the initial state of the timed automaton.

F is the set of Büchi states that have to be visited infinitely often.

Due to space limitation, the transformation of Until operator is not shown here. For details, please refer to the extended version of this paper [21]. The timed Eventually operator is summarized in Fig. 3 and 4. This is based on [14] with minor changes to match with our definition of TT. In these

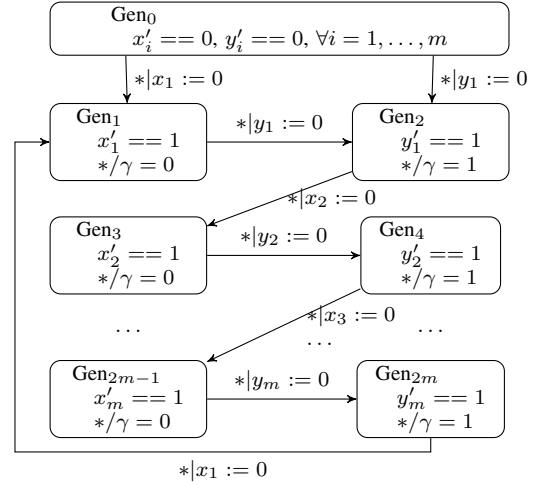


Fig. 3. The timed automaton for the generator part of $\diamond_I a$ for motion planning. $2m$ is the number of clocks required to store the states of the timed eventually (\diamond_I) operator. It is computed based on the interval I . Detailed computation and derivation can be found in [14]. x'_i represents the rate of the clock x_i . By setting the rate to be 0, we essentially deactivate the clock. The '*' symbol means that there is no value for that particular input, output or guard for that state. There are no Büchi states since the time is bounded.

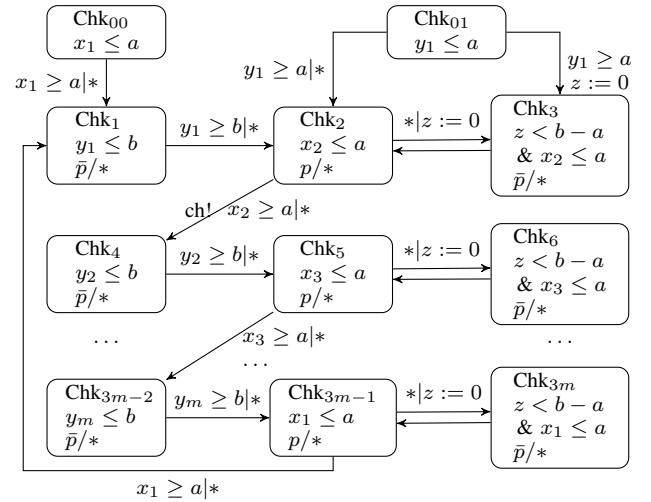


Fig. 4. The timed automaton for the checker part of $\diamond_I a$ for motion planning. $2m$ is the number of clocks required for the timed eventually (\diamond_I) operator. There are no Büchi states since the time is bounded.

figures, the inputs and outputs of the states are specified in the second line within the box of each state. $p\bar{q}$ means the inputs are $[1, 0]$ and \bar{p} means the inputs can be $[0, 1]$ or $[0, 0]$, and $\gamma = 1$ means the output is 1. Transitions are specified in the format of $g|r$.

The TT for timed eventually ($\diamond_I a$) is decomposed into two automata, the generator generates predictions of the future outputs of the system, while the checker verifies that the generated outputs actually fit the inputs. Detailed derivations and verifications of the models can be found in [14]. The composition between them is achieved through the shared clock variables. Additional synchronization ('ch!') is added in our case to determine the final satisfaction condition for the control synthesis. A finite time trajectory satisfies the

MITL, when the output signal of the generator automaton (Fig. 3) includes a pair of raising edge and falling edge verified by the checker automaton. The transition from Chk_2 to Chk_4 (Fig. 4) marks the exact time when such falling edge is verified. This guarantees that the time trajectory before the synchronization is a finite time trajectory that satisfies the MITL.

The composition of TT based on logic trees such as that of Fig. 2 is defined similar to [14] with some modifications to handle cases when logic nodes have two children, for example the until and conjunction operators.

Definition 3.2 (I/O Composition):

Let $\mathcal{A}_1^1 = (\Sigma_1^1, Q_1^1, \Gamma_1^1, C_1^1, \lambda_1^1, \gamma_1^1, I_1^1, \Delta_1^1, q_{10}^1, F_1^1)$, $\mathcal{A}_2^1 = (\Sigma_2^1, Q_2^1, \Gamma_2^1, C_2^1, \lambda_2^1, \gamma_2^1, I_2^1, \Delta_2^1, q_{20}^1, F_2^1)$ be the input sides of the automaton. If there is only one, then \mathcal{A}_1^1 is used. Let $\mathcal{A}^2 = (\Sigma^2, Q^2, \Gamma^2, C^2, \lambda^2, \gamma^2, I^2, \Delta^2, q_0^2, F^2)$ be the output side of the automaton. Because of the input output relationship between them, they should satisfy the condition that $[\Gamma_1^1, \Gamma_2^1] = \Sigma^2$. The composition is a new TT such that,

$$A = (\mathcal{A}_1^1, \mathcal{A}_2^1) \otimes (\mathcal{A}^2) = ([\Sigma_1^1, \Sigma_2^1], Q, \Gamma^2, C, \lambda, \gamma, I, \Delta, q_0, F)$$

where

$$Q = \{(q_1^1, q_2^1, q^2) \in Q_1^1 \times Q_2^1 \times Q^2, \\ s.t. (\gamma_1^1(q_1^1), \gamma_2^1(q_2^1)) = \lambda^2(q^2)\}$$

$$C = (C_1^1 \cup C_2^1 \cup C^2), \lambda(q_1^1, q_2^1, q^2) = [\lambda_1^1(q_1^1), \lambda_2^1(q_2^1)], \\ I_{(q_1^1, q_2^1, q^2)} = I_{(q_1^1, q_2^1)}^1 \cap I_{q^2}^2, q_0 = (q_{10}^1, q_{20}^1, q_0^2) \text{ and } F = F_1^1 \cap F_2^1 \cup F^2.$$

B. Path Synthesis using UPPAAL

The overall path synthesis framework is summarized as follows.

- First, the robot and the environments are abstracted to a timed automaton (TA) \mathcal{T}_{map} using cell decomposition, and the time to navigate from one cell to another is estimated based on the robot’s dynamics. For example Fig. 1.
- Second, MITL formula is translated to TT \mathcal{A} using the method described in the previous section.
- Then the product is taken of TT \mathcal{A} with the TA \mathcal{T}_{map} using the location label. For instance in Fig. 1 the product of $\text{pos1} : B$ will be taken with all states in TT that do not satisfy the predicate $p(a)$ but satisfy $p(b)$.
- The resulting timed automata are then automatically transformed to an UPPAAL [15] model with an additional satisfaction condition verifier. An initial state is chosen so that the output at that state is 1. Any finite trajectory which initiated from that state and satisfying the following conditions will satisfy the MITL specification. Firstly, it has to visit at least one of the Büchi accepting states, and secondly, it has to meet the acceptance condition for the timed eventually operator. To perform such a search in UPPAAL, a final state is added to allow transitions from any Büchi accepting state to itself. A verification automaton is created to

check the finite acceptance conditions for every timed eventually operator.

- An optimal timed path is then synthesized using the UPPAAL verification tool.

The implementations of the first and the second steps are based on the parsing and simplification functions of the `ltl2ba` tool [22] with additional capabilities to generate TT. We then use the generated TT to autogenerate a PYTHON script which constructs the UPPAAL model automatically through `PyUPPAAL`, a PYTHON interface to create and layout models for UPPAAL. The complete set of tools¹ is implemented in C to optimize the speed.

IV. CASE STUDY AND DISCUSSION

We demonstrate our framework for a simple environment and for some typical temporal logic formulas. Although our tool is not limited by the complexity of the environment, we use a simple environment to make the resulting timed automaton easy to visualize. Let us consider the timed automaton from the abstraction in Fig. 1 and the LTL formula given by the following,

$$\phi_1 = (\neg A \cup B) \wedge (\diamond A).$$

This specification requires the robot to visit the area B first and eventually visit A also. The resulting automaton based on the method in the previous section is as shown in Fig. 5. Each state corresponds to a product state between a state in \mathcal{T}_{map} and a state in TT \mathcal{A} . The Büchi accepting states are indicated by an additional b in their state names. We obtained the optimal path by first adding a final state and linking every accepting state to it, and then using UPPAAL to find one of the shortest paths that satisfies the condition “ $E \ll \text{final}$ ”. UPPAAL will then compute one fastest path in the timed automaton that goes to final state, provided such a path exists. If such a path exists, then it is a finite trajectory that satisfies the specifications. In this paper we are more interested in planning a path that satisfies some MITL constraint, so a finite time trajectory is a valid solution. The initial state of the automaton is loc0 which is the only state at pos0 that outputs 1. The optimal path is $\text{loc0} \rightarrow \text{loc2} \rightarrow \text{loc7} \rightarrow \text{loc6}_b$, in the product automaton. This means that the optimal way for a robot to satisfy the LTL constraint is to traverse the map in the following order, $\text{pos0} \rightarrow \text{pos1} : B \rightarrow \text{pos0} \rightarrow \text{pos3} : A$.

In the second test case, the environment stays the same and the requirement is captured in the MITL formula ϕ_2 :

$$\phi_2 = \square \diamond_{[0,2]} A$$

This requires the robot to perform a periodic survey of the area A every 2s. One part of the timed automaton is shown in Fig. 6. The full automaton can be found in the extended version of this paper in [21]. As we discussed earlier, if a synchronization signal (ch!) is sent, the falling edge for the output of the generator automaton is detected and verified. This marks the end of a finite trajectory that satisfies the

¹The tool is available on <https://github.com/yzh89/MITL2Timed>

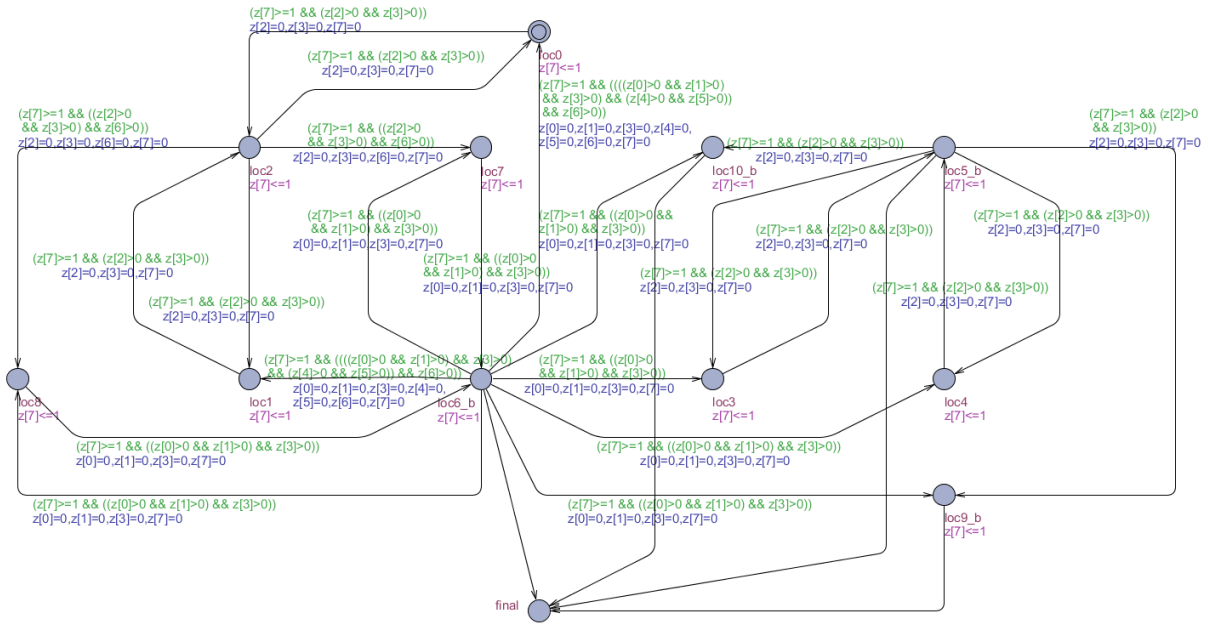


Fig. 5. The Resulting timed automaton in UPPAAL of ϕ_1 . The purple colored texts under the state names represent I . The green colored texts along the edges represent guard conditions, while the blue ones represent clock resets. The Büchi accepting states are represented by a subscript b in state names.

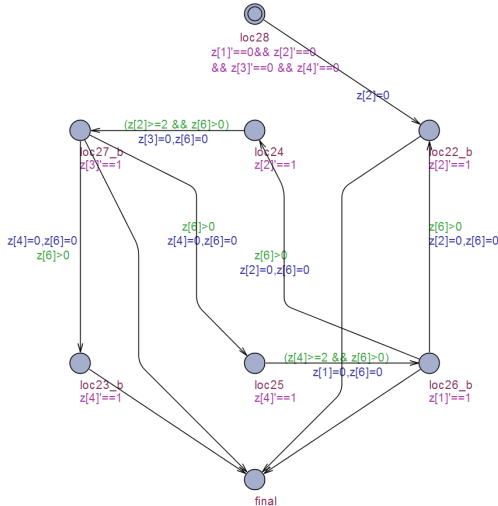


Fig. 6. The timed automaton of ϕ_2 corresponding to the generator of the timed eventually operator.

MITL constraints. Similar to the LTL case, we use UPPAAL to find one fastest path that leads to the final state. The optimal path in this case is $loc28 \rightarrow loc22_b$. This path corresponds to $(pos0, t \in [0, 1]) \rightarrow (pos3 : A, t \in [1, 2]) \rightarrow (pos0, t \in [2, 3])$ in physical space. Repeating this path will satisfy ϕ_2 .

All the computations are done on a computer with 3.4GHz processor and 8GB memory. Both of the previous examples require very small amount of time ($< 0.03s$). We also tested our implementation against various other complex environments and MITL formulas. Table I summarizes our results for complex systems and formulas. The map we demonstrated earlier is a 2x2 map (Fig. 1); we also examined

TABLE I
COMPUTATION TIME FOR TYPICAL MITL FORMULA

MITL Formula	Map Grid	Transformation Time	Number of TA Transitions	Synthesis Time
ϕ_1	2x2	$< 0.001s$	22	0.016s
ϕ_2	2x2	0.004s	69	0.018s
ϕ_3	2x2	0.40s	532	0.10s
ϕ_4	2x2	0.46s	681	0.12s
ϕ_1	4x4	0.004s	181	0.062s
ϕ_1	8x8	0.015s	886	0.21s
ϕ_2	8x8	0.015s	1795	0.32s

the cases for 4x4 and 8x8 grid maps. The other temporal logic formulas used are listed below. The time intervals in the formula are scaled according to the map size.

$$\phi_3 = \diamond_{[0,4]} A \wedge \diamond_{[0,4]} B$$

$$\phi_4 = \diamond_{[2,4]} A \wedge \diamond_{[0,2]} B$$

It can be seen from Table I that our algorithm performs very well with common MITL formulas and scales satisfactorily with the dimensions of the map.

V. CONTINUOUS TRAJECTORY GENERATION

In this section, we briefly describe the generation of a continuous trajectory from the discrete motion plan obtained from the timed automaton. Let us consider the nonholonomic dynamics of a unicycle car as given in (1).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = u \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix} + \omega \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (1)$$

where ω and u are the control inputs. It should be noted that the above nonholonomic dynamics are controllable and we assume no constraints on the control inputs at this point.

The outputs of the timed automaton are treated as the time-stamped way points for the robot to move. We have to ensure that the robot moves from one way point to the next with the given initial and final times, and at the same time, the trajectory should remain within the associated cells.

Since our environment is decomposed in rectangular cells, the robot only needs to move forward, turn right, turn left and make a U-turn. We synthesize a controller that can make the robot to perform these elementary motion segments within the given time.

For moving forward, the input ω is chosen to be 0 and the velocity u is tuned so that the robot reaches the final position in time. For turning left and turning right, ω is chosen to take positive and negative values respectively so that a circular arc is traversed. Similarly the U-turn is also implemented so that the robot performs the U-turn within a single cell.

Let us denote the state of the system at time t by the pair (q, t) i.e. $x(t) = q_1$, $y(t) = q_2$ and $\theta(t) = q_3$ where $q = [q_1, q_2, q_3]$. Then we have the following lemma on the optimality of the control inputs.

Lemma 5.1: *If $\bar{u}(t)$ and $\bar{\omega}(t)$, $t \in [0, 1]$, is a pair of control inputs, s.t. the dynamics moves from the state $(q_0, 0)$ to $(q_1, 1)$, then $u(t_0 + t) = \frac{1}{\lambda}\bar{u}(\frac{t}{\lambda})$ and $\omega(t_0 + t) = \frac{1}{\lambda}\bar{\omega}(\frac{t}{\lambda})$ move the system from (q_0, t_0) to $(q_1, t_0 + \lambda)$ for any $\lambda > 0$. Moreover, if \bar{u} and $\bar{\omega}$ move the system optimally, i.e.*

$$J(\bar{u}, \bar{\omega}) = \min_{u_1(\cdot), w_1(\cdot)} \int_0^1 [r_1 u_1^2(t) + r_2 w_1^2(t)] dt \quad (2)$$

then u and ω given above are also optimal for moving the system from (q_0, t_0) to $(q_1, t_0 + \lambda)$, i.e.

$$J_1(u, \omega) = \min_{u_1(\cdot), w_1(\cdot)} \int_{t_0}^{t_0+\lambda} [r_1 u_1^2(t) + r_2 w_1^2(t)] dt. \quad (3)$$

The proof can be found in the extended version of this paper [21].

Remark 5.2: Lemma 5.1 states that if the controls for elementary motions from initial time 0 to final time 1 are synthesized, then with proper scaling and shifting in time and scaling the magnitude, controls for any movement from any initial time to any final time can be synthesized without further solving any optimization problem.

VI. CONCLUSION

In this paper, we have presented a timed automaton based approach to generate a discrete plan for the robot to perform temporal tasks with finite time constraints. We implemented the algorithm in an efficient and generic way so that it can translate the time constraints and temporal specifications to timed automaton models in UPPAAL and synthesize the path accordingly. We then demonstrated our algorithm in grid type environments with different MITL formulas. We have considered grid type of environment for our case studies, but the approach can be generalized to most motion planning problems as long as the environment can be decomposed into cells. We also provide a brief overview on generating an optimal continuous trajectory from the discrete plan. For future works, we are considering to extend the work to include dynamic obstacles as well as multiagent systems.

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