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Distributed Solution of the Economic Dispatch Problem in Smart Grid Power Systems Framework with Delays

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Abstract— **We consider the Economic Dispatch Problem in power systems in a smart-grid architecture friendly environment. The problem is tackled with the use of multiple decentralized controllers that execute parallel distributed consensus algorithms. The scenario takes into account the presence of multiple time-varying communication delays.**

I. INTRODUCTION

Modern Energy Supply is typically a structure of interconnected power generation plants that independently produce power to serve a load over a common distribution network [16], [21], [17]. Since every power unit produces energy at some cost, a fundamental power optimization problem is the determination of the optimal combination of outputs of all generating units to minimize the total cost, while satisfying the load demand and operational constraints. This is the very well-known Economic Dispatch Problem (EDP).

Over the past years, many optimization methods for the EDP have been proposed in the literature. The conventional ones include the lambda iteration algorithm, or gradient-based search methods [16], [21], whereas modern heuristic optimization techniques are based on operational research and artificial intelligence concepts such as evolutionary algorithms [7], [9], simulated annealing [1], [20] artificial neural networks [2], [10], taboo search [8], [11] and particle swarm optimization techniques [3], [5].

Although the performance and applicability of economic dispatch has been improved by these optimization techniques, it is still essential to maintain a single control center that can access the state of the entire system. Indeed, all the aforementioned algorithms require operations at a central computing station that needs to have a priori knowledge of the entire network parameters. This centrally controlled framework may cause some performance limitations in the future power grid.

Since 1990, many electric utilities including both government- and private-owned electric utilities were liberated. This has had profound effects on the operation of electric systems where implemented, most of which is the economic value to the network operator. The EDP is a relevant procedure in the operation of a power system. The deregulation of the electric utilities has, therefore, led to research on a decentralized model of control where utilities, transmission system operators (TSO) and independent power producers (IPP) cooperate and compete using market and other mechanisms [21].

A. Smart-Grid Architecture

The next generation of power systems is expected to satisfy high standards of efficiency, resilience and reliability against cyber-attacks or natural disasters, improved integration of renewable energy resources and plug-in hybrid electrical vehicles. Such electrical grids, known as Smart-Grid, are designed to monitor, predict and intelligently respond to the behavior of all electric power suppliers and consumers connected to it in order to deliver such standards [4], [13].

Figure 1 is an abstract illustration of a Smart-Grid architecture. A necessary requirement towards this is the development of advanced control and communication technology both in the physical and the algorithmic layer. In a smart grid environment, the communication and measurement requires a multiagent systems (MAS) technology [13]. MAS are a computational system in which several agents cooperate to achieve a desired task. The performance of a MAS can be decided by the interactions among various agents. Agents cooperate to achieve more than if they act individually. Increasingly, MAS are the preferential choice for developing distributed systems such as the Smart-Grid [13]. The development of monitoring and measurement in Smart Grid with the use of MAS technology involves a combination of several agents working without human intervention, in collaboration pursuing assigned tasks to achieve the overall goal of the system.

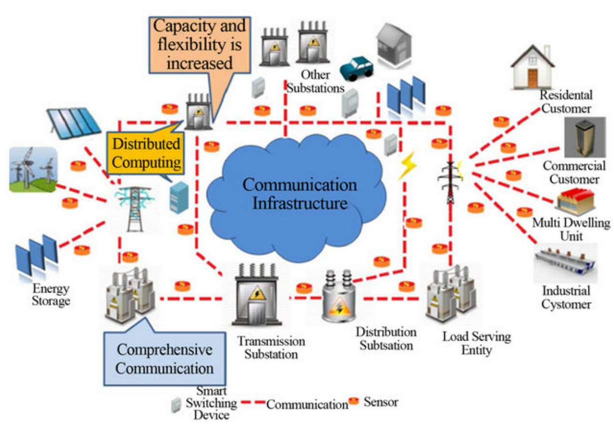

Fig. 1: A Smart-Grid electric network with multiple communication and control sensors.

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B. Consensus systems as computational mechanisms of distributed controllers.

A central algorithm in distributed systems is the so-called agreement or consensus algorithm under which a collection of autonomous agents engage in a dynamic convex averaging of a state of interest so that in the long run, they all obtain the same value. The history of consensus systems is very long as they are considered to be the underlying mathematical model for co-operative biological, social, robotic networks [6], [12], [14], [15].

C. Motivation, Contribution & Related Literature

The successful and reliable implementation of complex electrical networks such as the Smart-Grids, require advanced measurement and control methods that operate in a distributed way. Recent results in consensus systems [18] provide results that could be effectively applied to solve the fundamental EDP of an electric network in such a manner. The present work introduces and studies by theory and simulation a distributed solution of the EDP using a variation of lambda iteration algorithm [21], under the presence of arbitrary signal propagation delays.

1) Contribution: We consider a scenario of several power generators and loads connected to a common transmission network (see Figure 2). This is a grid enhanced with autonomous sensors, each of which controls at most one power generator and one power load. The sensors are connected to a common communication network and share certain information. The central characteristic of the communications is that they suffer from multiple time-varying delays. The network topology among the sensor is assumed time varying and complete, that is every agent communicates with each other even via arbitrary delayed signals. The objective is the solution of the EDP under the power generation constraints that each unit must meet and the delayed communication regime between the sensors. In particular every sensor will be designed to receive, process and transmit back information and will act both as a follower and as a leader in multiple computational levels so that the EDP is to be solved in a decentralized manner. The theoretical results will be supported with an illustrative example.

2) Related Works: The distributed solution of the EDP for power systems has been recently introduced in the literature [22], [23] where the use of averaging consensus schemes for solving the EDP problem are proposed with and without the presence of communication delays.

Our work differs on a number of points. At first, the model in the aforementioned works is primarily discrete and follows simplified average consensus schemes introduced in the early work of [15] both in ordinary and delay form. Those systems are "too symmetric", hence unrealistic, both in the communication rate and the imposed delays. In particular, the delayed case is treated in too much uniformity: each sensor receives the information from its neighbors under the same delay while it averages all the information with a delayed version of its own information. The working hypothesis is that the system dynamics evolve under both propagation and processing constant delays of identical magnitudes; a fairly unrealistic scenario. Moreover, the proposed algorithms solve the distributed EDP only in part. A leader sensor needs to be chosen so as to control the overall power mismatch and dynamically adjust the incremental cost value.

The present paper proposes a decentralized version of the lambda iteration algorithm. Each sensor controls a part of the electric grid, sends and receives information and executes multiple, simultaneous dynamic consensus iterations. This way it learns all the information needed to concur with the optimal operation point values that solve the EDP. In our scenario every sensor serves both as a leader and a follower in the network and in its utmost generality it needs to know the static parameters of the network, i.e. the connectivity weights and the delays each sensor operates under. However this is a knowledge on the communication level, no information on the transmission network is needed as the sensors, through the consensus scheme, learn the information (loads and generator powers) dynamically. The rate at which the agents learn depends on the communication parameters and of course it plays a vital role in the stability of the system.

II. Consensus Dynamics of Linear Systems with Delays

In this section, we will review recent results on the dynamics of linear consensus networks with delays. The discussion is drawn from [18]. Consider a set of $N < \infty$ autonomous agents each of which possess a value of interest, say x_i for $i = 1, \dots, N$ that shares and updates it dynamically so that $\mathbf{x}(t) = \big(x_1(t), \dots, x_N(t)\big)$ satisfies the following initial value problem

$$\dot{x}_i(t) = \sum_{j \neq i} a_{ij}(t)\big(x_j(t - \tau_{ij}(t)) - x_i(t)\big) \tag{1}$$

$i = 1, \dots, N$, $x_i(t) = \phi_i(t), t \in [t_0 - \tau(t_0), t_0]$ as initial data. Define the set

$$W_{t_0}(\boldsymbol{\phi}) := [\min_i \min_{s \in [t_0 - \tau, t_0]} \phi_i(s) - \min_i \max_{s \in [t_0 - \tau, t_0]} \phi_i(s)]$$

and take $|W_{t_0}(\boldsymbol{\phi})|$ to be its length. Set also the matrix $A = [a_{ij}(t)]$ with $a_{ii} \equiv 0$ is the, well-known from Graph Theory, adjacency matrix. Another important matrix is the degree matrix defined as $D(t) = \text{Diag}[\sum_j a_{ij}(t)]$. For the communication network we assume that it is fully connected and static but with time varying weights, i.e. $\forall i \neq j$, $a_{ij}(t) \Rightarrow a_{ij}(t) \in [\underline{\alpha}, \bar{\alpha}]$. Then for every $B > 0$ and $t \geq t_0$, $\int_t^{t+B} A(s)\, ds$ has every non-diagonal strictly positive and the matrix $P(t) = e^{-mB} I + \int_t^{t+B} e^{-m(t-s)}(mI - D(s) + A(s))\, ds$ is stochastic such that $\rho := \inf_{t \geq t_0} \min_{i,j} \sum_l \min\{p_{il}(t), p_{jl}(t)\} > N \min\left\{e^{-mB}, \frac{1-e^{-mB}}{m}\underline{\alpha}\right\} \in (0, 1)$

Theorem 2.1: [18] Consider the system (1) and its solution $\mathbf{x} = \mathbf{x}(t, t_0, \boldsymbol{\phi}),\ t \geq t_0$. If

1) $\sup_{t \geq 0} \max_{i,j} \tau_{ij}(t) = \tau < \infty$
2) for every $B > 0$ and all $t \geq t_0$, the matrix $\int_t^{t+B} A(s) ds$ has every non-diagonal element strictly positive,

then $\exists\, x_\infty \in W_{t_0}(\phi)$ such that

$$\max_i |x_i(t) - x_\infty| \leq \frac{|W_{t_0}(\phi)|}{1 - \rho e^{-(N-1)\bar{\alpha}\tau}} e^{-\gamma(t-t_0)} \tag{2}$$

where $\gamma = -\frac{\ln(1-\rho e^{-(N-1)\bar{\alpha}\tau})}{B+2\tau} > 0$.

A leader in a consensus network is an agent that affects the rest of the group, but it cannot be affected by it. In the presence of a leader, say agent 0 with state z_0 to satisfy a generic differential equation $\dot{z}_0(t) = g\big(t, z_0(t)\big)$ modeling possibly internal dynamics under the hypothesis

$$|z_0(t) - z_\infty| \leq Z e^{-\zeta(t-t_0)} \tag{3}$$

for some constants $Z, \zeta > 0$. The "leader-follower" system can be written as for $t \geq t_0$

$$\begin{cases} \dot{z}_0(t) = g\big(t, z_0(t)\big) \\ \dot{z}_i(t) = \sum_{j\neq i, i\neq 0} a_{ij}(t)\big(z_j(t - \tau_{ij}(t)) - z_i(t)\big) + \\ \qquad + a_{i0}(t)\big(z_0(t - \tau_{i0}(t)) - z_i(t)\big) \end{cases} \tag{4}$$

where $i = 1, \ldots, N$ and initial data

Theorem 2.2: [18] Consider the system (4) and its solution $\mathbf{z} = \mathbf{z}(t, t_0, \phi)$. Let the assumptions of Theorem 2.1 hold. Under (3) it holds that

$$|z_i(t) - z_\infty| \leq K_1 e^{-\gamma(t-t_0)} + K_2 \frac{e^{-\zeta(t-t_0)} - e^{-\gamma(t-t_0)}}{\zeta - \gamma}$$

for all $i = 1, \ldots, N$ and $K_1 = \frac{|W_{t_0}(\phi)|}{1-\rho e^{-(N-1)\bar{\alpha}\tau}}$, $K_2 = \frac{2Z\bar{\alpha}e^{\zeta\tau}}{1-\rho e^{-(N-1)\bar{\alpha}\tau}}$.

Remark 2.3: The aforementioned results hold also for simple (recurrent) connectivity regimes, as well. In this case the rate estimates change for the worse and they are beyond the scopes of our work. For more we refer to [18].

The presence of a leader does not alternate the qualitative behavior of the system other than the consensus point. This is the limit point of the leader. It is important to understand that the network eventually synchronizes to a constant value only when the leader converges to this value. Otherwise the followers, although never stop following the leader, they will not synchronize with it.

III. The Elementary Economic Dispatch Problem

A system of N power generating units, connected to a single bus bar serves a received electrical load P_{load} (Figure, 2). The input to each unit, shown as F_i represents the cost rate of the unit. The output of each unit P_i is the electrical power generated by that particular unit. The total cost rate of this system is the sum of the costs of each of the individual units. The essential constraint on the operation of this system is that the sum of the output powers must equal the load demand.

$$F_T = \sum_{i=1}^{N} F_i(P_i) \quad \text{s.t.} \quad \phi = 0 = P_{load} - \sum_{i=1}^{N} P_i$$

This is a constrained optimization problem that may be attacked formally using advanced calculus methods that involve the Lagrange function:

$$\mathcal{L} = F_T + \lambda\phi \tag{5}$$

The necessary conditions for an extreme value of the objective function result when we take the first derivative of $\mathcal{L}$ with respect to each independent variables and set the derivatives equal to 0:

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{dF_i(P_i)}{dP_i} - \lambda = 0 \tag{6}$$

Following [21] we will assume that the cost functions $F_i(P_i)$ are smooth and quadratic:

$$F_i(P_i) = \frac{1}{2}\chi_i P_i^2 + \psi_i P_i + \omega_i \tag{7}$$

for some strictly positive parameters χ_i, ψ_i, ω_i assumed to be known.

Together with (6) we must add the constraint that the sum of the power outputs must be equal to the power demanded by the load. All in all, we have the following systems of equations that it is necessary be satisfied in the optimal operation point:

$$i = 1, \ldots, N : \begin{cases} \frac{dF_i(P_i)}{P_i} = \lambda \\ \sum_i P_i = P_{load} \end{cases} \tag{8}$$

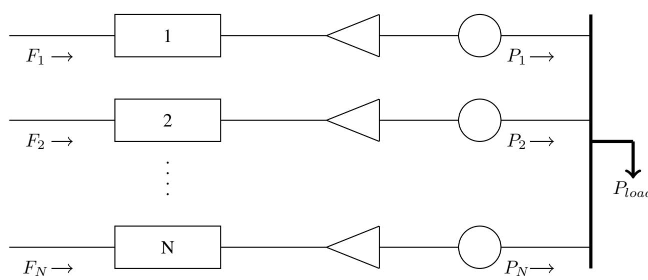

Fig. 2: N units committed to serve P_{load}. The schematics follow [21].

We characterize this problem as elementary because important parameters are ignored. In the discussion session we will explain interesting and more realistic generalizations within the theoretical context that is to be developed in the section to follow.

IV. Distributed Solution of the Loss-less EDP

Assume that at each thermal unit we have a controller that has full access to the parameters of its area of duty (load and generator) but it has limited information for the parameters of the other areas. The latter information is propagated through the communication network and it suffers from delays. In particular each controller i has

1) Instant information of the load in its section $P_{load}^{(i)}$ and transmission of it over the network. Here the i^{th} controller will be a leader of a consensus algorithm responsible to communicate this information to the rest of the controllers. We will adopt the notation $p_{(load,j)}^{(i)}$ the state of the follower $j \neq i$ on this algorithm. All in all, the vector

$$\mathbf{p}_{load}^{(i)} = \big(\ldots, p_{(load,i-1)}^{(i)}, p_{(load,i+1)}^{(i)}, \ldots\big)$$

symbolizes the states the sensor-followers, each of which seeks to learn the load the i^{th} controller transmits. These are all dynamic variables that follow a leader-follower consensus model. All in all we have N such models with possible different communication weights and delays of the type of Eq. (4) each of which has a leader i with $z_0 \equiv P_{load}^{(i)}$. We will assume that all, but the leader's, initial functions are set to zero. The vital characteristics of such a model are the lower and upper bounds of the communication weights and the maximum imposed delay. These are denoted as $\underline{\alpha}_i, \bar{\alpha}_i, \bar{\tau}_i$. The convergence of this model is guaranteed under a simple connectivity assumption as Remark 2.3 suggests.

2) Instant information on the power the i^{th} generator produces, denoted by $P_{gen}^{(i)}$. This will be dynamically updated and transmitted over the network satisfying the equation

$$P_{gen}^{(i)}(t) = \frac{\lambda_i(t) - \psi_i}{\chi_i}. \tag{9}$$

Here, the i^{th} controller communicates over the network the state $P_{gen}^{(i)}(t)$ with some delay.

3) Delayed received information of the signal $P_{gen}^{(j)}$ from all over the network. The i^{th} controller serves as a receiver of the generated power of the rest of the controllers with some delay.

A. The incremental cost algorithm

Each sensor chooses to update its state λ_i by averaging it with the rest of the sensors, as follows:

$$\begin{aligned}\dot{\lambda}_i(t) = &\sum_j w_{ij}(t)\big(\lambda_j(t-\tau_{ij}(t)) - \lambda_i(t)\big) + \\ &+ w_i^b\big(P_{load,i}^c(t) - P_{gen,i}^c(t)\big)\end{aligned} \tag{10}$$

were w_i^b is a coupling positive control parameter, $P_{load,i}^c(t)$ is the cumulative information of the load on the network, sensor i has at time t and $P_{gen,i}^c(t)$ is the cumulative information on the produced generator, sensor i has at time t. Using the notation above we deduce that

$$P_{load,i}^c(t) = P_{load}^{(i)} + \mathbf{1}^T \mathbf{p}_{load}^{(i)}(t) \tag{11}$$

$$P_{gen,i}^c(t) = P_{gen}^{(i)}(t) + \sum_{j\neq i} P_{gen}^{(j)}(t - \tau_{ij}(t)) \tag{12}$$

B. Analysis

The target of the sensors on a consensus value is $\lambda_i(t) \equiv \lambda_\infty$. From (7) we have the fixed point of $P_{gen,i}$

$$P_{gen,i}^\infty = \frac{\lambda_\infty - \psi_i}{\omega_i}.$$

Then the optimal operation point is

$$P_{gen}^\infty = \sum_{l=1}^N P_{gen,l}^\infty = P_{load} = \lambda_\infty \sum_{l=1}^N \frac{1}{\chi_l} - \sum_{l=1}^N \frac{\psi_l}{\chi_l} \tag{13}$$

Then limit consensus point, the sensors try to reach is:

$$\lambda_\infty = \frac{P_{load} + \sum_{l=1}^N \frac{\psi_l}{\chi_l}}{\sum_{l=1}^N \frac{1}{\chi_l}} \tag{14}$$

$$\begin{aligned}\dot{\lambda}_i(t) = &\sum_{j\neq i} w_{ij}(t)\big(\lambda_j(t-\tau_{ij}(t)) - \lambda_i(t)\big) + \\ &+ w_i^b\big(P_{load} - P_{gen,i}^c(t)\big) + w_i^b\big(P_{load,i}^c(t) - P_{load}\big)\end{aligned}$$

Now,

$$\begin{aligned}&\big(P_{load} - P_{gen,i}^c(t)\big) = \\ &= \frac{\lambda_\infty - \lambda_i(t)}{\chi_i} + \sum_{j\neq i}\big(P_{gen,j}^\infty - P_{gen}^{(i)}(t-\tau_{ij})\big) \\ &= \left(\sum_j \frac{1}{\chi_j}\right)\big(\lambda_\infty - \lambda_i(t)\big) + \sum_j \frac{1}{\chi_j}(\lambda_i(t) - \lambda_j(t-\tau_{ij}))\end{aligned}$$

so the consensus algorithm is written as

$$\begin{aligned}\dot{\lambda}_i(t) = &\sum_j \left(w_{ij} - \frac{w_i^b}{\chi_j}\right)\big(\lambda_j(t-\tau_{ij}) - \lambda_i(t)\big) + \\ &+ \left(\sum_j \frac{w_i^b}{\chi_j}\right)\big(\lambda_\infty - \lambda_i(t)\big) + g_i(t)\end{aligned} \tag{15}$$

where $g_i(t) = w_i^b\big(P_{load,i}^c(t) - P_{load}\big)$.

Eq. (15) is a perturbed consensus system with a virtual leader of constant value λ_∞. The weights of the new network $A = [a_{ij}]_{i,j\in\{0,N\}}$ with $a_{ii} = a_{0j} = 0$, $a_{i0} = \sum_{j=1}^N \frac{w_i^b(t)}{\chi_j}$ and $a_{ij} = w_{ij}(t) - \frac{w_i^b(t)}{\chi_j}$ elsewhere. Then using Theorem 2.2 we deduce that $\boldsymbol{\lambda}$ converges to the optimal economic point if there is an $c > 0$ such that

$$w_{ij}(t) - \frac{w_i^b(t)}{\chi_j} \geq c > 0 \qquad \forall i \neq j \tag{16}$$

the convergence of the algorithm occurs exponentially fast and the spread of the vector $\boldsymbol{\lambda}$, $\sup_{t\geq 0}(\min_i \lambda_i(t) - \max_i \lambda_i(t))$, is upper bounded by the constants K_1 and K_2 as they were defined in Theorem 2.2 as explicit functions of the systems' parameters. Also, Z is a constant determined by Theorem 2.1 as it is the consensus system under which each sensor communicates (learns) the load of the network acting as a leader (follower). Consequently $|Z| \leq \sum_{i=1}^{N-1} \frac{P_{load}^{(i)}}{1-\bar{\rho}_i e^{-(N-1)\bar{\alpha}_i\bar{\tau}_i}}$ as it was explained in the beginning of this section.

V. A Simulation Example

We will outline the previous analysis with an illustrative example taken from [21] (page 65). Here a network of $N = 3$ units generates power to serve a cumulative load of

$$P_{load} = 850 \text{ MW}$$

Each sensor is set to control part of this load and one generator as follows

1) Sensor 1: 200 MW of load and generates $P_{gen}^{(1)}$. The fuel cost function is

$$F_1(P_{gen}^{(1)}) = 561 + 7.92 P_{gen}^{(1)} + 0.001562 (P_{gen}^{(1)})^2$$

2) Sensor 2: 300 MW of load and generates $P_{gen}^{(2)}$. The fuel cost function is

$$F_2(P_{gen}^{(2)}) = 310 + 7.85P_{gen}^{(2)} + 0.00194(P_{gen}^{(2)})^2$$

3) Sensor 3: 350 MW of load and generates $P_{gen}^{(3)}$. The fuel cost function is

$$F_3(P_{gen}^{(3)}) = 78 + 7.97P_{gen}^{(3)} + 0.00482(P_{gen}^{(3)})^2$$

The units of F_i are in \$/hr. Using Eq. (6) and the condition $\sum_{i=1}^{3} P_{gen}^{(i)} = 850$ we derive the economic (consensus) point

$$\lambda_\infty = 9.148\$/\text{MWhr}$$

out of which the optimal generators

For the distributed solution of the above EDP we set $t_0 = 0$ and we consider the primary time-varying communication network

$$A = \begin{bmatrix} 0 & 0.5 & \frac{0.7+t^2}{t^2+1} \\ 1+\cos^2(1+t^2) & 0 & 6(1+e^{-t}) \\ 0.9 & \frac{0.7+t}{0.5+t} & 0 \end{bmatrix}$$

and the delays

$$T = \left[\tau_{ij}(t)\right] = \begin{bmatrix} 0 & 1 & 1 \\ 0.23 & 0 & 0.23 \\ 0.8\cos(16\pi t) & 0.8\cos(16\pi t) & 0 \end{bmatrix}$$

The secondary network involves the dynamic process of $P_{load}^{(i)}$ from $\mathbf{p}_{load}^{(i)}$. In particular:

Sensors 1 and 2 learn P_{load}^{3} with the state vector $(p_{load,1}^{(3)}, p_{load,2}^{(3)})$ under the network

$$\begin{cases} \frac{d}{dt}p_{load,1}^{(3)}(t) = 1.2\sin^2(t)\big(P_{load}^{(3)} - p_{load,1}^{(3)}(t)\big) \\ \frac{d}{dt}p_{load,2}^{(3)}(t) = 0.02\big(p_{load,1}^{(3)}(t-0.5) - p_{load,2}^{(3)}(t)\big) \end{cases}$$

Sensors 1 and 3 learn P_{load}^{2} with the state vector $(p_{load,1}^{(2)}, p_{load,3}^{(2)})$ under the network

$$\begin{cases} \frac{d}{dt}p_{load,1}^{(2)}(t) = 0.8\sin^2(2\pi t)\big(p_{load,3}^{(2)}(t-0.23) - p_{load,1}^{(2)}(t)\big) \\ \frac{d}{dt}p_{load,3}^{(2)}(t) = 0.6\sin^2(3\pi t)\big(P_{load}^{(2)} - p_{load,3}^{(2)}(t)\big) \end{cases}$$

Sensors 2 and 3 learn P_{load}^{1} with the state vector $(p_{load,2}^{(1)}, p_{load,3}^{(1)})$ under the network

$$\begin{cases} \frac{d}{dt}p_{load,2}^{(1)}(t) = 0.3\big(p_{load,3}^{(2)}(t-0.5) - p_{load,2}^{(1)}(t)\big) \\ \frac{d}{dt}p_{load,3}^{(1)}(t) = 1.7\big(P_{load}^{(1)} - p_{load,3}^{(1)}(t)\big) \end{cases}$$

Finally, the balance vector

$$(w_1^b, w_2^b, w_3^b)^T$$

is set to a common control constant $w_i^b \equiv w$. It is easy to see that the primary consensus system corresponds to a fully connected communication graph and the secondary consensus systems correspond to a simple connected graph. Also, all the delays are bounded. Then Theorems 2.1, 2.2 with Remark 2.3 apply and the aforementioned analysis holds with numerically calculated parameters $\zeta < 0.02$, $Z \leq 855$ $\underline{\alpha} = 0.35$, $\bar{\alpha} = 6$ and the largest delay is $\tau = 1$ and it is only a simple calculation to K_1 and K_2. The most important criterion however is Eq. (16). This imposes the smallness condition

$$w < w^* = 0.00194$$

Simulations are provided in Figure 3 for control values below and beyond w^*. We observe that whenever w is above this critical value the algorithm slows down or does not converge.

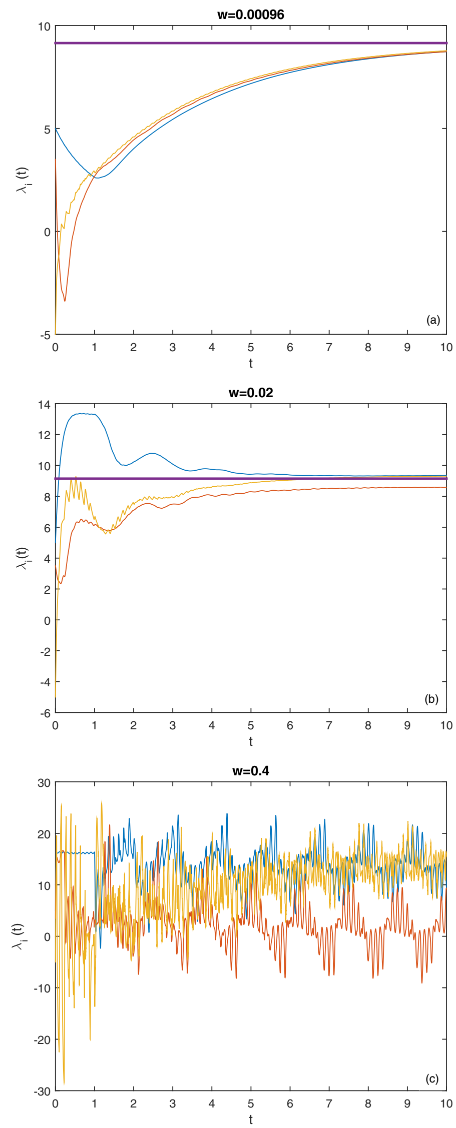

Fig. 3: Simulations run in MATLAB with the ddesd routine. The distributed incremental cost solutions $\lambda_i(t)$ is the consensus variable which appears to converge very fast for small w. (b) As w increases beyond w^* we still see oscillatory and slower convergence. (c) For w large the algorithm does not converge and for even larger values it diverges.

VI. Discussion

The solution of the EDP problem in a distributed manner is very important for the modern smart grid architectures. In this paper, we introduced a consensus based optimization algorithm that improves the existing ones [22], [23]. The theory develops a decentralized version of the lambda iteration algorithm with emphasizing on the communication network that controls the process. The optimal economic point is dynamically achieved via a communication network that suffers from multiple and complex delays. The present work is but a small step towards merging two very interesting fields of networked control systems: this of agreement dynamics and this of the modern electric power networks in the smart grid environment. Several things are yet to be addressed.

The elementary EDP basically we analyzed, neglects the fact that every power generator operates within limits. For the classical EDP one must substitute (8) with

$$i = 1, \dots, N : \begin{cases} \frac{dF_i}{P_i} = \lambda \\ P_{i,\min} \leq P_i \leq P_{i,\max} \\ \sum_i P_i = P_{load} \end{cases}$$

Within the developed setup, this issue can be tackled in two steps. The first is to recall that theory predicts explicit bounds on the difference of $\min_i \lambda_i - \max_i \lambda_i$. Since $P_{gen}^{(i)}$ can be expressed as a linear function of λ_i we are half way far from explicit bounds on the generated power $P_{gen}^{(i)}$. Indeed every sensor needs information on all the cost parameters α_j, β_j. Unless one is willing to set these parameters within universal standard bounds, these are information a sensor needs to learn from the network. Note that an important point is that for the EDP to have a solution, λ_∞ to be within the operation region of the generators. This is not always the case. Therefore a first extension is to develop algorithms that will, or attempt, to solve the EDP with given operating constraints.

A second problem with our approach is that we require an all-to-all connectivity even with arbitrary delays. It is very important to decentralize the architecture even further. It is not clear, however, how this could be achieved without critically destabilizing the algorithm. Even in the toy example of Section V, this complete communication regime could not suffice to stabilize solutions of the algorithm. This reveals the sensitivity of the consensus algorithm on perturbations and delays. Future research along this line would require a study on the connectivity regimes and how these affect the rate estimates and the stability bounds.

Another simplification assumption followed here, is that the power network is loss-less. If the energy network has energy losses, the EDP derives a slightly more complex Langrangian with an extra incremental cost value. Incorporating this factor on the dynamic algorithm is also a necessary step for designing more realistic theoretical algorithms. Given both $F_i(P_i)$ and the cost function of the transmission lines $F_{loss}(P_i)$ in quadratic form we conjecture that our theory could be adequately extended.

All the above observations unavoidably point to the final remark of our work: The form of the cost functions and the quadratic assumption. All the aforementioned questions can (and should) be repeated for more general cost functions. This is a fairly challenging issue even for the conventional methods [21].

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